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A Summary and Bibliography of Literature

by

St. Anthony Falls Hydraulic Laboratory
University of Minnesota

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P R E F A C E

Under Contract N6onr-246 between the University of Minnesota and the Office of Naval Research, Task Order 6 dated April 16, 1948 called for services to be rendered by the St. Anthony Falls Hydraulic Laboratory under the following specifications:

> "Conduct research on the detailed analysis of the basic mechanisms whereby atmospheric air is entrained in water by the relative motion of the two fluids. Such research shall consist of the primary phases as follows:
>
> 1. A review and analysis of the existing literature relating to the problem, including preparation of a selected bibliography with collected papers;
>
> 2. A correlation and extension of all existing theories to permit a firm analytical foundation for execution of the empirical investigations which can be treated in the laboratory; and
>
> 3. An extensive experimental laboratory program directed toward determining the mechanics of the phenomena and integrating the observations with the theory."

This report is a summary and bibliography of the subject as specified in the Contract. It has been written by Owen P. Lamb, who was assisted by Meir Pilch in library research and preparation of the abstracts, and by Lois Fosburgh in manuscript preparation. The manuscript was reviewed by Edward Silberman. The study was prepared under the supervision of Dr. Lorenz G. Straub, Director of the St. Anthony Falls Hydraulic Laboratory.

C O N T E N T S

SUMMARY OF LITERATURE

I. INTRODUCTION

The physical entrainment of a gas by a liquid and the flow of gas-liquid mixtures are phenomena commonly encountered in engineering practice, but avoided or arbitrarily compensated for in theoretical considerations and in design analysis. The progress toward a satisfactory explanation of these phenomena has been hampered by a lack of accurate experimental observations of entrained flows and by the complexity of the theoretical analysis when certain of the physical forces can no longer be neglected.

Flows of liquids with entrained gases may be roughly classified into those occurring in systems enclosed by solid boundaries such as conduits and those occurring in open channels where a boundary of the liquid stream is gaseous. Gas is often entrained in a liquid of a closed-conduit system in an easily understood manner, but the entrainment in an open channel remains a natural and imperfectly understood process. Although liquids and gases often flow as physical mixtures in closed-conduit systems, the only gas-liquid mixture of great practical importance in open-channel flows is that of water and air.

The published literature indicates that an awareness of the special problems involved in the design of structures and systems where entrained flows may be expected has been present throughout the last century. During the past 20 years the demand for a more comprehensive definition of these flows has intensified in proportion to the progressive limitation of uncertainty in other factors of hydraulic design.

This report is in partial fulfillment of Task Order 6, dated April 16, 1948, under terms of Contract N6onr-246 between the University of Minnesota and the Office of Naval Research. The specifications stated in part ". . . review the existing literature relating to the problem (of air entrainment), prepare a selected biography of collected papers . . .". A general limitation has been placed on the examination of the literature pertaining to flows of entrained fluids in vertical conduits and in open channels. In addition, there is included an abbreviated account of the related problems of the rate of rise of air bubbles in water and the unsteadiness of flow in open channels in as far as they may concern the study of entrained flows. The

review also includes selected reports of research and analysis relevant to determining the importance of surface-tension and viscosity forces in the flows of gas-entrained liquids.

The report is divided into two main sections. The first of these is intended to act as an introduction to the problems of entrained flows and is subdivided into the following groups: (1) Physical Properties of Gas-Liquid Mixtures, (2) Entrained Open-Channel Flows, (3) Entrained Closed-Conduit Flows, and (4) Experimental Methods. Important and reasonably substantiated results of the various investigations are listed in this section, although no common exposition was found that could be used for the purpose of comparing results. The second main section consists of a bibliography with abstracts of the pertinent literature.

II. PHYSICAL PROPERTIES OF GAS-LIQUID MIXTURES

A. General

When two liquids or a liquid and a gas are so completely mixed that no surface of separation is visible, the mixture can be treated as a single substance with unique properties of density and viscosity. If a liquid or a gas is dissolved in another liquid, the physical properties of the solution can be defined in terms of the physical properties and relative amounts of the original constituents. When the mixture of two liquids or a gas and a liquid assumes the entrained state, the two fluids are separated by a number of surfaces of macroscopic size and the combination can be no longer considered as an homogeneous fluid. The properties of density and viscosity vary from point to point in the fluid mixture, and to achieve a practical solution, the hydrodynamical analysis of motion of the mixture must include a consideration of the size and distribution of the entrained fluid masses.

The most common form of the entrained state is that in which bubbles or drops of entrained gas are dispersed in the entraining liquid. This is the case in closed-conduit systems, as well as in the portion of the flow below the violently agitated surface of open-channel flows subject to the occurrence of air entrainment.

Investigations of mixtures of two liquids in either the dissolved or the entrained state are not included in this review except in selected instances where a direct analogy may be drawn to the entrained mixture of a gas and a liquid. In low pressure regions of closed systems and in situations

where cavitation may occur, the gases dissolved in liquids are of importance. However, in the majority of entrainment considerations an accounting of the gas that passes over to the dissolved state would be a needless embellishment. Experimental and analytical investigations of gas-entrained liquids other than air-water mixtures are often treated in a general manner to allow the results to be directly applicable to air-water mixtures. Selected investigations of other gas-liquid mixtures are listed in the Bibliography.

B. Bubble Size and Shape

The shape that a volume of gas will assume when entrained in liquids is important to a thorough analysis of the flow of gas-liquid mixtures. In most experimental investigations, the relative amounts of gas and liquid are defined as a volume ratio of the gas to the total volume of gas and liquid. This ratio is, of course, a determining factor in the form of the mixture. A general classification by Versluys [41] designates mixtures with less than 50 per cent gas by volume as foam and those with more than 50 per cent gas by volume as mist. As there is no reason to believe that the form of the mixture changes abruptly when the concentration of gas passes through 50 per cent, the foam-mist terminology will be used in this report to distinguish the state where gas bubbles are surrounded by a liquid field (foam) from that where liquid droplets are surrounded by the gas (mist). The volume at which a transition from foam to mist will occur is dependent upon both the tension at the interface of the gas and liquid in question and the degree of agitation of the fluid mass. Observation of a highly turbulent entrained open-channel flow will reveal both the foam and the mist condition.

The experimental and analytical investigations of gas bubbles ascending in a liquid column and of liquid droplets falling in a gaseous field are more numerous and, in some respects, more advanced than the studies of the concurrent flow of the gas and the liquid as a mixture. By reference to these studies, it is possible to establish a first approximation to the extent of the influences of bubble size, distribution, shape, settling-out velocity, bubble growth by coalescence, and relative velocities near the bubble surface on the flow of entrained mixtures. Although individually these influences may be small, collectively they must account for several of the measured differences between the liquid flows containing many small volumes of gas and the flows of homogeneous liquids.

Bubble size and shape are both governed by the apparent tension at the interface between the gas and the liquid. Since this tension is a function of the curvature of the interface, as well as of the relative properties of the fluids involved, the bubble tends to more closely approach a spherical shape as the bubble size is decreased. In effect, the bubble becomes stronger and is better able to withstand rupturing inertia and shear forces with the decrease in size. The mean size and shape of entrained gas masses would seem to be dependent upon the balance between the disruptive forces in the flow and the resistance of the gas-liquid interface to these forces, rather than upon the method by which the gas became entrained.

It has been shown by Miyagi [23] and others that a marked change in the rising velocity and motion of bubbles takes place when a critical radius of the bubble is exceeded. The buoyant force of the gaseous mass is dominant at the larger radii, which causes a natural sorting of sizes in entrained flowing mixtures. Pekeris [27] has summarized the results of several investigations on the rate of rise of various sized bubbles in connection with his report on the diffusion of air bubbles.

Wilde and Moore [42] have investigated the slippage near the gas-liquid interface for bubbles of various size, and Miyagi [23] has proposed an equation designed to define the amount of liquid carried by a moving bubble of given size. The term "slippage" has been commonly used in the literature to denote the relative velocity of gas and liquid portions of an entrained fluid. An actual slip at a fluid interface would be impossible. Definite limitations to the degree of extrapolation that might be used in applying data on the motion of single gas bubbles to the hydrodynamical analysis of gas-liquid flows are mentioned in the report by Gosline [9] on his experiments on the vertical flow of gas-liquid mixtures.

C. Surface Tension

When there are many surfaces separating a gas from a liquid, the phenomena of surface tension become important. A modification of the hydrodynamical equations of motion has been suggested by Korteweg [20] for the case of a fluid in which the variation of density from point to point is so rapid that the phenomena of capillarity must be considered. He has also considered the case of a thin transition layer between the solid and vapor state of a fluid, which has a possible bearing on the entraining mechanism through a free

surface in open-channel flow. The equations and analysis suggested by Korteweg have not found extensive use in later investigations.

Ohnesorge [24] and Haenlein [10], among others, have studied the dissolution of liquid jets and the formation of drops from a jet discharging into the air. Their methods and results are applicable in an evaluation of the ratio of the surface-tension forces to the inertia and viscous forces of an entraining liquid. Entrainment at the lip of an overflow pipe (a condition that has been investigated in studies of drop inlets, tank drains, and inlet structures where "gulping" may occur) is dependent upon surface-tension forces throughout the lower range of heads over the lip of the inlet. In investigations of this type, the chief importance to the study of air-entrained flows is in the evaluation of the quantity of air introduced into the closed system under observation. Kalinske [16] made quantity measurements of the air entering this type of structure.

The importance of surface tension in the determination of bubble size has been mentioned in the preceding section, but its effect on the shape of the bubbles might be mentioned again at this point. The property of causing the bubble shape to be nearly spherical could be of great value in simplifying the analysis of the gas-liquid mixtures and in developing instruments to measure entrained flows. Properties of suspensions of both uniform and random-sized spheres have been treated quite extensively in analogous fields of electricity and physics in a manner that promises to be useful in gas-liquid investigations.

A direct analysis of surface-tension forces readily applicable to problems of air entrainment could not be found in the literature. The absence of a basic explanation of the surface-tension phenomena is particularly noticeable because there is no abundance of experimental and analytical data that could be expeditiously applied to special problems where surface-tension forces are significant.

D. Viscosity

The inability to use the commonly accepted parameters of Reynolds number and Froude number directly in the classification of air-entrained flows is a serious deterrent to the comprehensive investigation of these flows. The measurement of the viscosity of a gas-liquid mixture by ordinary methods is not feasible and special methods that would render reliable results have not

been perfected. Similarly, velocities and length dimensions that define the
inertia forces in these dimensionless numbers are not as easily defined or
measured as they would be in a flow of a nonentrained homogeneous liquid. The
expression Reynolds number of the flow would need an inherent compensation
for the changes of density throughout the cross section of entrained open-
channel flows.

The viscosity of a gas-liquid mixture can be defined in a closed
tube or system where a direct comparison with homogeneous flows can be made,
but this method cannot be treated independently of the tube that was used for
the comparison. The assumption of uniform distribution of the gas throughout
the liquid is inherent in the definition. For the limited conditions of the
flow resistance of gas-oil mixtures through vertical pipes, Uren [38] concluded
that the density and viscosity of gas-oil mixtures vary as logarithmic func-
tions of the gas-oil ratio.

An analytical derivation for the viscosity of a suspension of solid
spherical particles, which was later modified to include ellipsoidal shapes,
was further expanded by Taylor [35] to include the case of the suspension of
one fluid in another. The assumptions of no slip at the surface of the drop,
small spherical shape, and continuity of tangential stress at the surface were
necessary to this derivation which is not easily applied to practical cases
in engineering design.

III.   ENTRAINED CLOSED-CONDUIT FLOWS

A. General

The flow of entrained mixtures in closed-conduits differs in many
respects from air-water flows in open channels. Entraining processes in closed
conduits, normally due to either an air leak at a section below atmospheric
pressure or to a designed injection apparatus, are not as difficult to explain
and to predict as the natural entrainment of air through the free surface of
a high velocity stream. Because of the large density difference between
liquids and their entrained gases, the gas distribution in a pipe depends to
a large extent on the orientation of the pipe with respect to the horizontal.
In the limiting case of a vertical conduit it is often safe to assume a uni-
form distribution across the section. Until measurements of velocity distri-
butions in pipes and channels with established entrained flows are available,

the analysis of similarity to distribution is limited practically to an assumption of similarity to distributions encountered in the flow of homogeneous fluids. An additional factor accounting for the resistance at the roughened surface of a high velocity open-channel flow is sometimes included. Despite the many differences in conduit and channel flow, the more thorough investigations conducted with controlled closed-conduit systems undoubtedly offer much of value to the less advanced study of self-aeration in high velocity flumes.

B. Application of Vertical-Conduit Measurements

The study of the rate of rise of single gas bubbles in a liquid may be applied directly to practical problems such as the determination of the extent and the duration of the noise interference field behind the screws of a ship, and the cleansing of air from liquid in recirculating systems. However, in applications where the flow persists in an extremely turbulent state as in entrained open-channel flows or in gas-lift operations, the large range of bubble sizes and the frequent size changes due to coalescence and rupture of bubbles lessen the value of a direct extrapolation of single bubble data. Single bubble data are as yet the only criteria that are available for approximating an expression defining the settling-out velocities and buoyant forces, allowing an initial determination of the natural sorting of bubble sizes evidenced in channel flow.

The gas-lift equipment used to bring oil and natural gas to the surface in petroleum fields usually deals with far greater ratios of gas to liquid than are found in open-channel entrainment studies. Most of the work done in these systems is by virtue of the expansion and temperature changes of the gaseous masses, factors that are usually of little consequence in open-channel investigations. The four critical flow conditions of the vertical gas lift, namely, (1) point of no flow due to insufficient air, (2) point of maximum efficiency, (3) point of maximum liquid flow, and (4) point of no flow due to an excess of air, define the limiting conditions of the relevancy of vertical-conduit data to open-channel considerations. Between the point of maximum efficiency and the point of maximum liquid flow the highly compressed mixtures at the bottom of the eduction tube have volume ratios of gas to liquid of the same order of magnitude as found in naturally entrained channels. Since the gas exists as bubbles in the liquid, it is feasible to examine possible

relationships of the data obtained in these systems to the general subject of air entrainment.

The experiments of Wilde and Moore [42] regarding slippage fall into the classification of those data that might be conveniently used to formulate relations between variables not readily determined by use of open-channel measurements. By testing mixtures of air with five different liquids in a vertical tube, they concluded that slippage was a function of tube diameter, as well as of the density and surface tension of the liquid, but that it was independent of the viscosity of the liquid. Although many factors of greater importance to open-channel entrainment than slippage remain indeterminate in the analysis of vertical-conduit experiments, it would seem likely that an evaluation of the frictional losses in flowing nonhomogeneous fluids could be conducted more easily in closed systems. However, most conduit investigations recorded in report form failed to measure all of the variables that would allow an isolation of frictional effects directly relevant to open-channel studies. In the future study of isolated variables in closed-conduit flows it is important to remember that there are several possible flow conditions in the conduits where the distribution of gas differs appreciably from the distributions observed in open channels.

C. Application of Hydraulic Machinery Data to Entrained Flows

Among the sources of data of use in a general analysis of air entrainment phenomena are specialized investigations of hydraulic machinery. These investigations include studies of the machinery which exhibits less efficient operation because of entrainment and machinery which depends for its basic operation upon the work done on or by the entrained gas. Processes which utilize a hydraulic method of entrapping and cleansing gas from points of collection in closed systems may also be included in this category.

In conventional types of turbomachinery, the inclusion of a sizeable quantity of air with the incoming liquid would result in decreased efficiency of the installation. Quantitative observations of this decrease in efficiency were made by Glassgold [8] on a small Pelton wheel. In the practical design of conventional pumps and turbines, however, air entrainment need not be considered, as it is usually possible to prevent its occurrence.

Machinery depending upon the action of the entrained air for its operation includes hydraulic air compressors and gas-lift pumps. In the former,

work is done by a descending column of water on the entrained gas bubbles; in the latter, the bubble expansion assists the upward liquid flow. There are considerable data available on both of these systems, although the hydraulic air compressor has found very little usage in recent years. These data would be useful as a larger scale control on laboratory investigations where entrained flows are directed both upward and downward in vertical conduits. In this way, an expression for the frictional losses in a length of conduit could be formed independently of an evaluation of buoyancy effects. Chapter XIV in the text by Peele [26] presents elementary theory and references to hydraulic air compressor installations, while the report by Owens [25] includes elementary theory of the gas-lift pump and a bibliography of more advanced investigations. Although the reports on these installations are directly concerned with gas-liquid mixtures, the experimental work rarely produced data that might be used to evaluate physical properties of these mixtures. Thus the design studies on these machines apparently offer little toward a general analysis, but they have much to gain from any relationships concerning gas-liquid flows proved in future investigations.

Kalinske and Robertson [17] investigated the method of cleaning air pockets from conduits by a hydraulic jump. An interesting critical value between the ability of the jump to entrain air and the ability of the downstream flow to carry the air away has been developed in their work. Relative merits of the undular jump and scavenging surface waves were evaluated in a discussion of this same paper. Their verification of the momentum principle for determining hydraulic jump characteristics when air is considerably entrained is of interest to the design of steep gradient flumes and spillways in which both air entrainment and the dissipation of energy are important factors. It is unfortunate that their study was not extended to include an analysis of losses downstream of the jump where an entrained mixture was flowing, because the authors had both a visual access to the flow and an established method of measuring the separate discharge of air and water.

IV. ENTRAINED OPEN-CHANNEL FLOWS

A. General

Air entrainment in open-channel flow produces an increase in depth of flow, a decrease in capacity of a given flume, and undetermined changes in

mean velocity, velocity distribution, and shear distribution. The mechanism by which the air is entrained has been regarded as analogous to sediment transportation, wave forms at the boundary between superposed liquid streams, density layer occurrences, turbulent mixing and diffusion phenomena, and a basic unsteadiness in high velocity gravity flows. Disagreement on such basic points as the relative velocities of entrained streams and otherwise similar nonentrained streams can be found in the literature. The limitation of the number of theories concerning the entraining process and the entrained flow must depend upon reliable observations of the flow and an evaluation of the assumptions upon which the theories are based.

Experimental investigations of entrained open-channel flows, including both laboratory and field measurements, were conducted chiefly in flumes of rectangular section. An essentially two-dimensional character of the flow was either assumed during the course of the measurements or was later assumed in order to simplify the computations involved in interpreting the collected data. In all but two of the relevant studies the analysis was limited to bulk values of air discharge, water discharge, and velocity.

B. Experimental and Analytical Investigations

The inability of each investigator to fit his experimental data to any generally acceptable function is apparent upon perusal of the functions proposed. The empirical mode of attack used in most of the studies rendered varying relationships between slope, flow depth, velocity, and air content. These functions defined selected portions of the data of that study but did not agree well with measurements of other investigations, and in most cases inadequately defined the remaining range of the data obtained on the same channel reach with the same instruments. Consequently, even in the most comprehensive recorded investigations there are many qualifying limitations placed on the proposed functions.

A discussion of each proposed expression relating the variables of open-channel entrainment would necessarily be a discussion of the differences between them. In the absence of a rational analysis of the problem to serve as a yardstick, a discussion of this type could do little more than illustrate the difficulties involved in the general study. A general classification of the functions evolved from the experimental programs illustrates a tendency to extrapolate the range of the commonly used formulas for low gradient open

channels to apply to channels with higher slopes. A computed hydraulic radius based on the perimeter of the aerated section and area of the water mass has been used by Streeter [34] to bring the coefficient into approximate agreement with the roughness factors commonly used for various channel-lining materials. In his analysis of the data, Hall [12] utilizes the common Manning equation and illustrates its application to practical design works, but he assumes the ordinary n values for the particular lining material valid when used with the area and perimeter of the aerated section. The use of the computed hydraulic radius as a length parameter for Froude numbers was designated as erroneous by DeLapp in a discussion of the same paper because this procedure introduced the channel breadth into the Froude expression.

Streeter proposed adding a term to the Manning equation to account for air resistance and the energy expended by the flow in maintaining motion in the air stream above the flow. This step was taken to defend the belief that the velocity of the entrained stream must necessarily be slower than that of a nonentrained, but otherwise similar, flow. Inherent in his analysis were the assumptions that the resistance due to the air boundary was proportional to the square of the velocity and that this resistance would aid in balancing any tendency toward an increase in velocity due to an increase in hydraulic radius with greater air content of the flow. The agreement of his derived functions with the experimental data was postulated on the probability of the velocities recorded by the salt-velocity method, being maximum center velocities of the stream rather than average values affected by observed sinuosity and other distracting factors.

By observing initial entrainment along the sides of steep gradient flows and then an entrainment across the whole surface, Lane [21] first made the qualitative proposal that the onset of entrainment was dependent upon the state of the turbulent boundary layer. At the channel sides the turbulent layer is at the water surface from the start, which accounts for the early entrainment of air at these locations. As the flow progresses, the entrained areas on the sides grow larger until they span the breadth of the flume. In flows that are relatively shallow in relation to their breadth, the boundary layer from the channel bottom reaches the surface first to mark the line of initial entrainment which is quite apparent on large spillways. This line is moved farther down the face of the spillway as the depth of flow over the crest is increased in proportion to the additional growth necessary for the

boundary layer to reach the surface. These observations can be readily accepted because it is now generally agreed that turbulence is the mechanism by which air is held in the flow.

Knapp [12] suggested the probability that a situation similar to the interfacial mixing of density currents could be applied to air entrainment problems. This approach would restrict the mixing to the upper layers of the flow, leaving the bottom layers relatively unaffected. The mixing zone would form a density flow which would move at a velocity intermediate between the air and water velocities and would serve as a buffer to inhibit further mixing.

With data which included vertical traverses of air concentration on the flume centerline, DeLapp [4] concluded that the distribution of air throughout the depth of flow was a turbulent diffusion phenomenon and proposed a formula based on the assumption of a constant diffusion coefficient that fit his data quite well. The absence of reliable data on the velocity distribution, that could be coupled with the extensive measurements of air concentration, prevented a wider analysis of the whole problem at that time. An empirical expression giving the mean percentage of air in the flow as a function of the slope did not satisfy the data for the entire range of slopes tested by DeLapp. The same difficulty was observed by Ehrenberger [6] , who found it necessary to alter the constants in his expression to fit the data obtained in the laboratory and on the Ruetz wasteway.

The wave forms observed on steep gradient flumes present an interesting study that probably has a close connection with the study of air entrainment and the mechanism of entrainment. Several references concerning this subject are listed in the Bibliography, although wave studies have not been incorporated in an air entrainment program to any extent.

With the exceptions of the report by DeLapp on distribution of air and the several reports recording mean velocity measurements on large flumes, there are very few data in the literature that can serve as a useful guide for the development of a general theory that will answer practical demands. The need for a rational basis of interpreting air entrainment phenomena and experimentally verifying functional relationship to the extent that they become acceptable to the designing engineer, is manifest at this time if progress in the use of steep gradient flumes, wasteways, and high overflow spillways is to continue.

V. EXPERIMENTAL METHODS

A. General

The analysis of fluid flow in open channels or conduits depends upon experimental measurements for guidance in development of the theory and for confirmation of the developed theory as an accurate explanation of the physical occurrences. In many cases a specific fluid system can be expeditiously designed and constructed by limiting the experimental investigation to the problems at hand. However, faulty measurements and inadequate instrumentation lessen the value of the experimental efforts of an investigator whether he is seeking a general trend or a particular solution. Significant effects of the variables involved, possible scale effects of a model study, and an accurate evaluation of constants in dimensionless expressions can be masked easily if the measurements of length, velocity, density, viscosity, surface tension, and other fluid properties can be only roughly approximated.

Instrumentation of fluid flow systems has been the subject of much lengthy and painstaking research with the result that satisfactory methods for the evaluation of most common properties of a homogeneous fluid have been evolved. It is unfortunate that this wealth of experience cannot be directly applied to the instrumentation of a study of a nonhomogeneous fluid such as a gas-entrained liquid. The lack of satisfactory means for obtaining experimental data is undoubtedly a major deterrent to the development of the study of air entrainment and must be adequately accounted for before a comprehensive experimental and analytical program can be made to progress efficiently.

B. Experimental Channels

When an experimental study is confined to laboratory facilities and when, as is often the case, a versatile testing model cannot be built to a scale as large as will eventually be utilized, the question of scale effect is raised. A restrictive wall effect on the free movement of the gas bubbles in smaller conduit models has been investigated quite extensively by Miyagi [23] and others. As initial entrainment in a conduit system is not usually dependent upon model size and in most cases is alien to the ensuing flow of the entrained mixtures, the consideration of sufficient reach to achieve entrainment pertinent in open-channel work has no counterpart in the conduit studies of gas-liquid mixtures.

It is apparent that a model spillway, with the length ratios ordinarily encountered in laboratory practice, exhibits no indication of the entrainment phenomena, although the prototype may have "white water" at all operating stages. Escande [7] has analytically demonstrated that similitude of the air entrainment phenomena is impossible in systems comprising a free surface subjected to atmospheric pressure. The condition for similarity demands the same reduced pressure at the free surface as must be observed for the similitude of cavitation phenomena in model studies. Smetana [33] qualitatively considered the problem of similitude in the particular open-channel phenomena of hydraulic jumps when air is considerably intermixed in the flow, but a number of his observations are valid for a more uniform open-channel flow as well. Schmidt [30] has listed considerations dealing with the dimensional similitude and general form of the functions of gas-liquid mixtures. Although the special cases of rise velocities of given-diameter, single bubbles and diffusion of fuel jets injected into compressed gases by cylindrical nozzles were mentioned, his paper was presented primarily to recall the principles of dimensional similitude and to demonstrate that a judicious combination of the important variables may greatly expedite experimental work when gas-liquid mixtures are being studied.

The inability to produce entrainment with a small-scale, open-channel model and the failure of application of ordinary similitude considerations in air-entrained systems, having free surfaces at atmospheric pressure, necessitate the establishment of a general relationship of the many variables involved that can be easily applied to engineering design. With this in mind, several experimental programs were initiated by various agencies and investigators dealing with flumes subject to air entrainment and controlled to some extent during the testing processes. Some of these investigations were made on large existent flumes with water discharge as the only controlled variable, while others were made on variable-slope, laboratory flumes. Considering the observations recorded in the paper by Hall [12] and in the USBR Laboratory Report by Thomas [36], it is difficult to determine whether a final condition of air entrainment had been reached for most of the slopes of the large outdoor flumes with which they were concerned.

In a variable-slope, laboratory flume 50 ft in length, DeLapp [4] reasonably established the fact that the flow at the lower end of the St. Anthony Falls flume had attained terminal velocity and had reached a maximal

condition of entrainment; that is, it was freeing as much air as it was en-
training.   To attain this condition in such a short reach, it was necessary
to jet the flow at the flume inlet at the approximate terminal velocity for
the set slope and discharge.   Bureau of Reclamation tests with a laboratory
channel did not attempt to achieve a measurable degree of entrainment, but
rather were concerned with development of their velocity measuring instruments.
Using a laboratory channel with lengths from 18 to 52.5 ft, varying with de-
creasing slopes, Ehrenberger [6] found that he had achieved a maximal entrain-
ment for all but the highest slopes and shortest reaches.   He also had found
it necessary to introduce the flow at a high initial velocity to achieve en-
trainment.   Tests by Ehrenberger [6] and by Innerebner [14] on the Ruetz
wasteway in Austria were reported to be representative of the condition where
maximum air concentration had been achieved at the lower end of the flume.
In the longer reaches of the flumes discussed by Thomas and Hall, the air
content in the flow had probably attained a uniform condition with all pre-
vious effects of acceleration and retardation eliminated in time for the final
flow of a given slope to become established.

In analyzing the problems of the designer of hydraulic structures,
it becomes apparent that many flumes and spillways which may concern him never
attain terminal velocity or air-content capacity at all flow stages for the
subject slopes.   Most steep, sloped structures will have an accelerating flow
over a large part or all of the total reach and have only a fractional part
of the maximum air concentration probable in a very long length.   This actual
air concentration must be determined from the point where the stream begins
entraining air, and both the remaining reach available for entrainment and the
acceleration of the fluid must be considered.   Hence, an ideal exposition of
the problem would demand a straight, variable-slope flume with a symmetrical,
well-designed inlet through which water initially at rest would be drawn.   A
length of flume necessary to achieve full acceleration and air concentration
of the flow over an unbroken bottom slope would also be needed.   A report by
Durand [5] proposes a function establishing the reach necessary to attain a
given velocity on steep slopes.

Proceeding from the work of DeLapp and Ehrenberger, the length of a
laboratory channel designed to study the onset of entrainment and growth of
entrainment to a maximum air concentration should be at least 30 ft.   An upper
limit of from 50 to 60 ft in length would probably be dictated by the size and

space requirements of a fully variable-slope flume complete with operating mechanism and instrumentation appendages. This length would not permit a full acceleration of the fluid from rest if reasonable depths of flow at the lower end of the testing channel are to be realized, thereby necessitating a fairly high initial velocity at the inlet.

The designed breadth of a testing channel of this type would be a function of the calculated depth of flow desired for the testing program. It is important that the effect of a boundary layer growing from the bottom of the flume should be independent of the entrainment along the channel sides for some central portion of the flow. The channel used by DeLapp was 1 ft in breadth and that used by Ehrenberger was 0.82 ft wide. Because of the absence of complete horizontal and vertical traverses of velocity and air concentration in these channels, it is not possible to use these data to isolate side effects from the entrainment process.

The choice of length and breadth of a proposed experimental testing flume, as well as the flume inlet design, the selection and evaluation of boundary roughness, the method of water supply and control, and the provision of convenient appendages for instrumentation must be carefully considered and approximated in as reliable a manner as possible if an apparatus suitable for the study of air entrainment in open channels is to be evolved. In their relation to air entrainment, the factors of flow acceleration and retardation will probably have to be studied in judiciously selected existing flumes after the basic processes of entrainment and flow of entrained mixtures are better understood.

C. Velocity Measurements

The inclusion of a considerable and unknown volume of air in flowing water precludes the possibility of simply determining the mean velocity of flow by dividing the quantity of water flow by the cross-sectional area of the flowing stream.

Pressure-type velocity instruments are useless if the density of the mixture cannot be determined accurately because of the presence of the air. Most other normally used methods encounter similar difficulties in a non-homogeneous fluid.

Previous air entrainment investigations have resorted to methods which determine velocity by observing the time necessary for a marked mass to traverse a definite reach of channel, or have calculated mean velocity after the respective quantities of air and water flowing are determined. Most of these methods result in mean velocities of the fluid stream rather than comprehensive velocity traverses that could be used in a more complete analysis.

Perhaps the most intensive effort to develop a useable technique was that of the USBR, discussed in the paper by Thomas [36]. Utilizing the salt-velocity method, they conducted laboratory tests first on an existing model and then on a specially built flume with a bottom slope of 45°. Several useable data concerning electrode size, channel coverage, spacing, and interpretable oscillographic readings were obtained from these laboratory tests. Later developments of the laboratory technique were evolved during the velocity measurement runs on the existing steep gradient, full-scale flumes with which they were primarily interested. These exploratory efforts of the USBR, with application of salt-velocity methods to open channels, afford a valuable set of data for any project concerned with the experimental investigation of the air entrainment phenomena.

Among the data recorded in the paper by Hall 12 were observations on three flumes of the Pacific Gas and Electric Company. Velocity measurements in these flumes were accomplished by visually observing the passage of a cloud of dye, either flourescein or potassium permanganate, and timing this dye mass over a measured reach of channel. Several difficulties, attributed to the inability of accurately defining the boundary of this color mass, were mentioned. Error by the observer was lessened to some extent by using averages of the times recorded by several observers at the lower end of the measured reach where the dye masses had diffused considerably in the entrained flow.

Ehrenberger [6] assumed a straight line variation of velocity between a point obtained near the bottom of the flow, where a conventional Pitot tube could be used with reasonable accuracy because of the low air concentration, and a point obtained near the flow surface by utilizing the surface-float principle. Aside from any error this assumption may have introduced, the reliability of the data dependent upon a measurement of local velocity by the surface-float principle remains questionable because the float will have a variable draft dependent upon the degree of air concentration and hence the

density near the surface.   Instances were mentioned of conditions where the float intermittently dragged on the bottom because of the lessened density.

Although the instrument used by DeLapp [4] was primarily designed to obtain air concentrations at points in the flow cross section, it was possible to obtain velocity measurements at these same points by dividing the sum of the measured quantities of air and water by the cross-sectional area of the aspirator tube. Conversion of this instrument to a velocity measuring device would have the important advantage of affording a means of obtaining traverses of the velocity in a cross section of the flow.

Approximations to the mean velocity of flow have also been obtained with the assumption that the presence of the air has a negligible effect on the velocity.   This extrapolation of the velocity functions derived and accepted for homogeneous flows on slight slopes has neither been proved nor disproved to a degree that would permit an accurately defined limitation of the designed velocities when air entrainment is probable on steep gradient slopes.

The measurement of mean velocity in closed-conduit experiments is ordinarily accomplished by solution of the continuity equation permissible because the cross-sectional area of the flow is rigidly defined and discharge of the gas and the liquid can usually be measured separately at some point before or after the testing section.   Any satisfactory method developed to obtain velocity traverses would be a boon to closed-conduit studies of gas-liquid mixtures, as well as open-channel studies, although an added complexity to some foreseeable methods must be considered in steeply inclined conduits where slippage between the gas and liquid portions assumes importance.

D.   Measurement of Air Concentration

The flow sampler used extensively in air entrainment tests at the St. Anthony Falls Hydraulic Laboratory and described in the report by DeLapp [4] is the only instrument cited in available literature that has been satisfactorily used to obtain air concentration at points in the cross section independently of velocity measurements.   Using traverses with a conventional Pitot tube and interpolated values of velocity (previous section), Ehrenberger [6] calculated air concentration from the deficiency between the velocity head obtained experimentally and that expected if water free of air were flowing at the observed point.   Most other experimental studies presented bulk values of entrainment obtained from and dependent upon the measurements of velocity and depth of flow.

The apparatus used by DeLapp consisted essentially of a method of drawing off a filament of the flow by aspiration methods, without causing a disturbing curvature of the streamlines, and then a separate metering of the air and water portions of the withdrawn filament. Careful operation of this apparatus resulted in an accurate determination of air concentrations from 5 per cent to 80 per cent by volume. Values above and below these limits were subject to large errors because of size limitations of the measuring apparatus. An important disadvantage of this method is the large expenditure of time, averaging about 30 minutes a reading, necessary to obtain satisfactory results.

By reference to methods used in processing industries for measuring relative amounts of the constituents of mixtures without separating them, it would seem that a more satisfactory method of obtaining air concentration could be evolved. Possible approaches would include measurement of changes in conductive, dielectric, spectral or refractive properties of the mixture.

E. Flow Depth Measurements

The measurement of flow depth, usually a simple and straightforward process in a lined channel, presents a formidable problem when air entrainment occurs. Since depth measurements determine the cross-sectional area of the flow and the length parameters for Froude and Reynolds numbers and are a basic factor of interest in designs where freeboard and energy dissipation are important, it is imperative that an accurate and simple method of obtaining these dimensions be developed.

In the absence of a clearly defined surface, it became necessary for various investigators to determine the flow depth in approximate manners which imposed severe limitations on the accuracy of their resulting impirical functions. The most common group of these approximations involved the expedient use of common stage measuring equipment, such as point gages, in an arbitrary manner. The stage where mist just obscured the point of the gage from the observer's sight and the stage where an attached electrical circuit was complete 50 per cent of the time are representative examples of defined surfaces determined by point gage equipment. Other noteworthy definitions of mean flow depth are based on stages where arbitrary forces are exerted on impact elements of special design, as well as on depths obtained by extrapolation of experimental data to the 100 per cent air contration level.

F.   Surface Observations

In the study of the entraining mechanism it is desirable to slow the violent surface agitation and apparent wave forms to a velocity that can be observed in detail visually.   High speed motion picture photography would seem to be the best method of realizing this objective.   Thomas [36] mentioned a series of photographs taken by the USBR with a standard 16 mm motion picture camera at a rate of 64 frames per second and then projected at a rate of 16 frames per second.   The resultant slowed motion clearly revealed the existence of characteristic but undefined waves or protuberances at the entraining surface.   A slower motion, attainable by use of a much faster rate of photographing the surface, was recommended for further study.

A N N O T A T E D B I B L I O G R A P H Y

(1) Arnold, H. D. "Limitations Imposed by Slip and Inertia Terms Upon Stokes' Law for Motion of Spheres Through Liquids." PHILOSOPHICAL MAGAZINE, Series 6, Vol. 22, pp. 277-775. 1911.

        The author discusses departures from Stokes' law due to the velocity of immersed spheres and to the slip at the interface of sphere and liquid, and records experimental values of these differences. Experiments were conducted with several spheres in various liquids to determine the limits of validity of Stokes' law in accord with the assumptions upon which the law is derived. It was found that:

      1. Ladenburg's correction for the effect of the walls of the fall-tube is independent of the viscosity and is valid for spheres of radii not greater than one-tenth the radius of the tube.

      2. Stokes' law with the above correction is valid for the less viscous liquids.

      3. The inertia terms begin to have an appreciable effect when the radius of the spheres is about 0.6 of the critical radius.

      4. The theoretically derived formula involving the coefficient of sliding friction is valid for the case of complete surface slip.

      5. The slip at the surface of a gas bubble rising through a liquid may be either approximately zero or infinite. This can be explained from the film-forming properties of the liquids, which may also explain irregularities of previous experiments on the rise of air bubbles.

      6. A consideration of the erosion at the surface of an air bubble leads to the possibility of a new method of determining the relative spheres of molecular attraction in liquids.

(2) Burke, S. P. and Plummer, W. B. "Suspension of Macroscopic Particles in A turbulent Gas Stream." JOURNAL OF INDUSTRIAL ENGINEERING CHEMISTRY, Vol. 20, pp. 1200-04. 1928.

        The constant of the equation for the force acting between a macroscopic spherical particle and a turbulent gas stream moving past it has been determined as

$$F = 0.00084 \, \rho \, v^2 r^2$$

all units being in the cgs system of units (F = grams). The value of the

constant is slightly higher for irregular coal or coke particles when the radius r is determined as the radius of a sphere of equal volume. The relationship was established by determining the loss in weight of various particles hanging in vertical air streams of known velocity, and confirmed by observation of the velocities required to suspend free particles in a vertical air stream. The available information (covering the cases of streamline as well as semiturbulent conditions about the particle) and the limits of application of the formula are discussed.

(3) Cromer, S. and Huntington, R. L. "Visual Studies of the Flow of Air-Water Mixtures in a Vertical Pipe." TRANSACTIONS OF THE AMERICAN INSTITUTE OF MINING ENGINEERS, Petroleum Division, Vol. 136, pp. 79-90. 1940.

Quantitative experimental data have been obtained on the flow of air-water mixtures in a 2-in. diameter vertical tube 90 ft high. Visual observations were made at six equally spaced intervals through short pyrex glass sections. It was found that four distinct types of flow may take place in the vertical tube, depending upon the air-water ratio. These four types may be descriptively called (1) piston flow, (2) annular ring flow, (3) mixed flow in the foam condition, and (4) mixed flow in the mist condition.

(4) DeLapp, W. W. THE HIGH VELOCITY FLOW OF WATER IN A SMALL RECTANGULAR CHANNEL. Ph. D. Thesis, University of Minnesota, 1947. 44 pages.

Tests were conducted for the determination of the amount of air entrained in the flow in a rectangular channel 12 in. wide, 10 in. deep and about 50 ft long, at 10 slopes ranging from 1° to 44°, and at discharges ranging from 0.15 to 10 cfs. Three different roughnesses were used in each case. The measurements were made by means of a specially designed flow sampler. The following results were obtained:

1. Air entrainment began at a channel slope of approximately 4° at a discharge of 2.5 cfs, regardless of channel roughness.

2. Turbulence in the flow distributed the air in accordance with the relationship

$$y - y_m = m \log c$$

at the channel centerline in a section normal to the bottom, where y is the depth, y_m is the mean depth of flow, m is the slope of the air distribution curve on a semilogarithmic plot, and c is the volume of air in a unit volume of air-water mixture.

3. The value of the turbulent diffusion coefficient was essentially constant over the section of the centerline for each discharge. This is a corollary of result two.

4. The mean percentage of air in the flow increased approximately in a linear manner with channel slope, and was as great as 50 per cent at the highest slope.

5. The rate of discharge, hydraulic radius, and surface roughness all were found to have very minor effects on the percentage of air in the flow.

6. The Manning formula was found to be suitable for high velocity flow as a first approximation. The use of normal values of Manning's n led to no appreciable error in computing the magnitude of the mean velocity of the flow; the greatest error occurred for rough surfaces and high slopes.

7. Entrainment of air resulted in substantial increases in the depth of flow at high slopes.

8. A close analogy was found between the entrainment of air and the transportation of suspended sediment.

(5) Durand, W. F. "The Flow of Water in Channels Under Steep Gradients." With discussion by G. H. Keulegan and H. N. Eaton, TRANSACTIONS OF THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, pp. 9-14. January 1940. Without discussion, ENGINEERING, pp. 120-22. July 1939.

This paper presents a mathematical stepwise solution to the problem of determining the magnitude of the flow velocity after a given length of run or after a given period of time of flow in steep channels. The magnitude of this velocity can be computed from the known initial conditions and from a known or an assumed value of a coefficient β, which relates the resistance to flow at a given point in the run or at a given instant of time to the square of the velocity. It is found that $\beta \sim \dfrac{\lambda}{R_m w}$ where λ is the friction coefficient for unit velocity, R_m is the mean hydraulic radius and w is the weight of a unit volume of the fluid. The values of the coefficient β must be determined experimentally.

(6) Ehrenberger, R. "Wasserbewegung in steilen Rinnen (Schusstennen) mit besonder Berücksichtigung der Selbstbelüftung," (Flow in Steep Chutes, with Special Reference to Self-Aeration). ÖSTERREICHISCHER INGENIEUR - und ARCHITEKTVEREIN, No. 15/16 and 17/18, 1926. 23 pages. Translated by E. F. Wilsey, U. S. Bureau of Reclamation.

Experimental investigations of air entrainment phenomena were made in a smooth rectangular wood channel 0.82 ft wide at five different slopes ranging from about $9°$ to $37°$ with the horizontal. Four rates of flow ranging from 0.35 to 1.57 cfs were used at each slope. A conventional Pitot tube was used to determine velocities near the bottom of the channel where little air was found to be present. An estimate of the surface velocity was obtained by means of a float, and intermediate velocities not subject to direct measurements were obtained by linear interpolation. The measurement of the apparent velocity head at these intermediate points by means of the Pitot tube indicated the percentage of air in the flow past these points.

On the basis of these experiments, it was concluded that self-aeration of the flow begins at a velocity varying from 6.6 to 9.8 fps in smooth wooden flumes, and that the mean velocity of flow for such conditions is lower than the velocity computed by the usual formulas for nonaerated flow at the same depth and slope. The author proposed the formula

$$V = 97\, R^{0.52}\, S^{0.40}$$

for determining the velocity of the aerated flow when the hydraulic radius $\underline{R}$ (area of flow section divided by perimeter of wetted boundary) and the channel slope $\underline{S}$ are known. He also suggested two formulas for determining the percentage of water in the flowing mixture:

$$P = \frac{42}{R^{0.05}\, S^{0.26}} \quad \text{for } S < 0.476$$

and

$$P = \frac{30}{R^{0.05}\, S^{0.74}} \quad \text{for } S > 0.476$$

The author examined data obtained from measurements made on the Ruetz wasteway in Austria and by making certain assumptions and adjustments,

he was able to show fair agreement between these and his laboratory re-
sults. However, because of the empirical nature of the derived equations
and the relatively small number of tests on which they are based, these
equations can be regarded only as rough approximations.

(7) Escande, L. "Sur la Similitude des phénomènes d'entraînement d'air par
l'eau en mouvement," (On the Similitude of Phenomena of Air Entrainment
by Water in Motion). COMPTES RENDUS, Vol. 209, pp. 626-27. 1939.

In continuation of previous work (Similitude of Vortices, COMP-
TES RENDUS, Vol. 194, 1932, pp. 1048-51), the author shows that the prin-
ciple of similitude is theoretically not applicable to systems comprising
a free surface subjected to atmospheric pressure, even if it be assumed
that perturbations attributable to viscosity are negligible due to tur-
bulence, and if the effect of surface tension is not taken into account.
The case of the entrainment of air by a stream of water is discussed in
illustration of this contention.

(8) Glassgold, I. THE EFFECT OF ENTRAINED AIR ON THE JET OF A PELTON WATER
WHEEL. M. S. Thesis, Towne Scientific School, University of Pennsyl-
vania. 1947. 112 pages.

This thesis contains performance curves of a small Pelton wheel
with and without entrained air and discusses the design of the needle
valve of the turbine system.

(9) Gosline, J. E. "Experiments on the Vertical Flow of Gas-Liquid Mixtures
Through Vertical Pipes." TRANSACTIONS OF THE AMERICAN INSTITUTE OF MIN-
ING ENGINEERS, Petroleum Division, Vol. 118, pp. 56-70. 1936.

Experiments were conducted to determine the feasibility of ap-
plying data on the motion of single gas bubbles ascending through viscous
liquids to the hydrodynamic analysis of vertical flow of gas-liquid mix-
tures in pipes, which necessitates an evaluation of the relative velocity
between gas and liquid phases. The results indicated that such data can-
not be applied to the complex mixture flow. Experiments on stationary
mixtures formed with air and three liquids of widely different physical

characteristics indicated that these characteristics of the liquids had little effect upon the mean velocity of the air relative to the liquid. Experiments with a small, all-glass, gas-lift apparatus, using three colorless liquids and air, indicated two distinct types of admixture of gas and liquid in the riser pipe. Low ratios of air to liquid were accompanied by an extremely turbulent, unsteady type of flow. At higher ratios of air to liquid, the liquid passed up through the tube as an annulus in contact with the walls, while the air passed up the central portion of the tube. Highest efficiency occurred in connection with the unsteady, turbulent type of flow.

A theory was developed for the annular ring type of flow and data (obtained by traversing the cross section of the pipe with a small impact tube) were used to calculate the relationship between drag coefficients and absolute air velocities for three liquids used.

(10) Haenlein, A. "Über den Zerfall Eines Flüssigkeitsstrahles," (On the Dissolution of a Jet of Liquid). FORSCHUNG AUF DEM GEBEITE DES INGENIEUR-WESENS, Vol. 2, pp. 139-49. 1931.

This article describes the procedure and results of an experimental investigation conducted for the purpose of visualizing the process of dissolution of liquid jets of different density, viscosity, and surface tension at various diameters and velocities of the jet. The following liquids were used: water, gas, oil, glycerine, and castor oil. Details are given of the apparatus for producing a jet of 0.1 to 1.0 mm in diameter at velocities ranging from 2 to 70 mps.

Photographs of the jet show the following four characteristic forms of dissolution of the jet: formation of drops without influence of air, formation of drops with influence of air, wave formation, and shattering. In the case of drop formation when the air has no influence, the time for dissolution of the jet is found to be independent of the jet velocity. This dissolution time was found to differ for different diameters and liquids. A regular relation between the dissolution time and the jet diameter can be established on the basis of similitude.

(11) Halbronn, G. "Vitesses limites comparées des bulles d'air dans l'eau et des gouttes d'eau dans l'air," (Comparison between Terminal Velocities of Air Bubbles in Water and Drops of Water in Air). LA HOUILLE BLANCHE, pp. 60-61. January-February 1948.

> The limitations of the application of Stokes' law to the motion of fluid bubbles in a viscous medium are discussed and the factors causing this limitation are analyzed. The relative validity of the results of several investigations is discussed and curves are given showing the relation between the terminal velocities of drops and bubbles and their radius.

(12) Hall, L. S. "Open Channel Flow at High Velocities," (Paper I of "A Symposium on Entrainment of Air in Flowing Water"). With discussions by W. Delapp, R. T. Knapp, J. H. Douma, J. C. Stevens, D. C. McConaughy, T. J. Corwin, and L. S. Hall, TRANSACTIONS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS, pp. 1394-434 and 1393-513. 1943.

> Experiments were made on several large chutes of different slopes at a relatively small range of discharges. The mean velocity of flow was determined by observing the course of a cloud of dye introduced at the upstream end of the flume. On the basis of these tests, the author concluded that the entrainment of air results in higher velocities than would occur for water alone, and that the ratio of air to water in the mixture is directly proportional to the square of the mean velocity divided by the hydraulic radius, with the latter computed on the basis of the area occupied by the water alone. A factor of proportionality was determined for each of the channels tested; this factor varied over a three-fold range, its value apparently depending upon the channel shape and roughness.

(13) Hall, L. S. "The Influence of Air Entrainment on Flow in Steep Chutes." UNIVERSITY OF IOWA STUDIES IN ENGINEERING, BULLETIN 31, pp. 298-314. 1946.

> This article presents a theoretical analysis of the results of experimental measurements of velocities and entrainment of air in actual chute structures. This analysis leads to the development of formulas for flowing water in steep chutes, in which allowance is made for the effect of air entrainment. An example is given of the application of these formulas to the design of steep chutes.

(14) Innerebner, K. "Overflow Channels from Surge Tanks." WORLD POWER CONFERENCE, Vol. 2, pp. 481-85. 1924.

Data on air entrainment were obtained from measurements in the overflow channel of the Ruetz wasteway in Austria. The measurements were taken in the upper gradient (about 38°) with the help of floats. The measured stretch was straight and about 290 ft long. The real water discharge was measured at the overflow of the water chamber by a staff gage, and the quantity of the water and air mixture was calculated as the products of the observed velocities and cross sections.

(15) Jeffreys, Harold. "The Flow of Water in an Inclined Channel of Rectangular Section." PHILOSOPHICAL MAGAZINE, Series 6, Vol. 49, No. 293, pp. 793-807. 1925.

A theory has been constructed to account for certain traveling waves observed in steeply inclined conduits. It appears that the uniform turbulent flow of a stream, with a plane free surface, becomes unstable when the mean slope exceeds 1 in 100, and that it is then replaced by a series of bores traveling faster than the water. Observational evidence is qualitatively consistent with the theory, but quantitatively several points remain to be tested.

(16) Kalinske, A. A. "Hydraulics of Vertical Drains and Overflow Pipes." UNIVERSITY OF IOWA STUDIES IN ENGINEERING, BULLETIN 26, pp. 26-40. 1939-1940.

Experimental data on the hydraulics and pneumatics of three sizes of vertical overflow and drain pipes when flowing partly full are presented. It was found that the discharge varied as the square root of the pipe diameter and as the square of the head. For flow partly full the head-discharge relationship was independent of the pipe length. For a given pipe size and head, the overflow pipe discharged slightly more water than the drain pipe.

The critical head above which the pipe starts to flow full was found to depend upon the pipe size, pipe length, and type of entrance. An analysis was made which permitted the prediction of this critical head for any size and length of overflow or drain pipe. The predicted values

agreed with the experimental data quite well. The ratio of the rate at which air was drawn down with the water, to the water discharge, was a maximum at a relatively low head. The maximum rate of air flow for any given condition occurred at a head considerably less than the critical head. For any pipe size and water discharge, the air flow increased with pipe length. However, the increase became less for longer pipes, indicating that the air inflow would tend to be independent of the pipe length for very long pipes.

(17) Kalinske, A. A. and Robertson, J. M. "Air Entrainment in Closed Conduit Flow." With discussions by K. R. Kennison, C. E. Kindsvater, J. W. Johnson, and A. A. Kalinske and J. M. Robertson. (Paper II of "A Symposium on Entrainment of Air in Flowing Water"), TRANSACTIONS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS, Vol. 108, pp. 1435-1516. 1943.

The general problem treated is the study of air entrainment by flowing water in pipes, particularly with reference to the removal of air pockets from water supply lines. The ability of the flow to carry off all of the air entrained by a hydraulic jump at the air pocket is defined by a critical value of the Froude number of the flow approaching the jump for any slope of pipe and relative depth of flow in the air pocket. Quantitative data on the rate of air flow through a hydraulic jump and on characteristics of the hydraulic jump in closed sloping circular conduits are presented.

(18) Keulegan, G. H. and Patterson, G. W. "A Criterion for Instability of Flow in Steep Channels." TRANSACTIONS OF THE AMERICAN GEOPHYSICAL UNION, Part II, pp. 594-96. 1940.

Proceeding from the basic definition that the flow in an open channel is unstable if conditions are such that a disturbance of the free surface increases in magnitude as it moves downstream, the authors present a theoretical analysis of the phenomena involved and develop a simple criterion for instability. This criterion is derived from Boussinesq's equation for the velocity of propagation of a volume-element of a wave, and from Manning's formula for the variation of channel resistance with depth.

(19) Keulegan, G. H. and Patterson, G. W. "Effect of Turbulence and Channel Slope on Translation Waves." JOURNAL OF RESEARCH OF THE NATIONAL BUREAU OF STANDARDS, Research Paper RP1544, Vol. 30, pp. 461-512. June 1943.

This paper is the second of a series dealing with the motion of flood waves and other waves of translation in open channels. The first paper considered waves controlled solely by inertia forces; while this is an analysis of the combined effects of turbulent friction and inertia. The basic equation of motion for gradually varied unsteady flow in prismatic channels is derived from fundamental principles. The effect of the velocity distribution in the original undisturbed current on the motion of short waves is investigated, and the effects of wave height, curvature of profile, and fluid friction on the celerity of a wave-volume element are analyzed in detail. The deformation of a straight sloping front and the change of height of an abrupt wave front are treated. Special emphasis is placed on disturbances of negligible curvature; and practical methods of handling engineering problems arising in connection with the operation of locks or hydroelectric canals are given.

(20) Korteweg, D. J. "Sur la Forme que Prennent les Équations du Mouvement des Fluides si l'on Tient Compte des Forces Capillaires Causées par des Variations de Densite Considérable Mais Continues, et sur la Theorie de la Capillarité dans l'Hypothèse d'une Variation Continue de la Densité," (The Form Taken by the Equations of Motion of Fluids if the Capillary Forces Caused by Variation of Density are Regarded as Considerable but Continuous, and the Theory of Capillarity Based on the Hypothesis of Continuous Variation of the Density). ARCHIVES NÉERLANDAISES DES SCIENCES EXACTES ET NATURELLES, Series 2, Vol. 6, pp. 1-27. 1901.

The author presents a mathematical development of the equations of motion of compressible and incompressible viscous fluids under consideration of continuous capillary forces produced by variation of density. A geometrical interpretation of the capillary terms in the equations is given. Several particular cases are analyzed: dissolution in a compressible fluid of variable concentration or of a mixture of two liquids of variable proportions; capillary equilibrium in the case of horizontal layers and concentric spherical layers of equal density; variation of normal pressure along a trajectory orthogonal to surfaces of equal density; and a thin transition layer separating two homogeneous phases.

(21) Lane, E. W. "Recent Studies of Flow Conditions in Steep Chutes." ENGI-NEERING NEWS-RECORD, pp. 5-7. January 1936.

Selected observations, by the author, of flow in steep chutes indicate an increasing entrainment of air after the turbulent boundary layer growth until the turbulent, entrained zone extends throughout the entire flow section. The effect of convex and concave vertical curves in steep chutes and their proper design are discussed. Effects of non-uniform width of channel and the peculiar resulting wave action are analyzed. Finally, analysis is made of friction losses at high velocities, based on observations of flow in the Uncompahgre flume in Colorado. Roughness values, determined by the Manning and Kutter formulas, are plotted against discharge. No definite conclusions are made concerning roughness values of the flumes observed.

(22) Lauffer, H. "Pressure, Energy and Flow Conditions in Channels with High Gradients." Technical Memorandum No. 527, Bureau of Reclamation, Denver, Colorado, 1936. Translation by D. P. Barnes of "Druck, Energie und Fliesszustand in Gerinnen mit grossem Gefalle," WASSERKRAFT UND WASSER-WIRTSCHAFT, Vol. 30, pp. 78-82. 1935.

This article presents the following theoretical and experimental demonstration: For parallel flow with high gradients, the pressure head in the interior of the liquid is no longer equal to the vertical distance from the surface, but is significantly smaller. It follows that the q-line (q is the discharge per unit width), as well as the dynamic capacity (hydrostatic pressure plus transfer of momentum through cross section in unit time) are dependent upon the slope. The surface profile corresponding to the minimum dynamic capacity is both the boundary between shooting and streaming flow and the surface profile for maximum discharge. The Froude number for this condition of flow can vary between zero and one, depending upon the slope; that is, $Fr = \sqrt{\cos\psi}$, where ψ is the angle of inclination with the horizontal.

(23) Miyagi, Otogoro. "The Motion of an Air Bubble Rising in Water." PHILO-SOPHICAL MAGAZINE, Series 6, No. 295, Vol. 50, pp. 112-40. July 1925.

Air bubbles of various sizes moving up in still water are carefully treated experimentally and then theoretically. Their terminal velocities are determined in relation to their sizes, and the changes of

their shapes during their motion are investigated. The mass of water carried up with a moving bubble and the resistance to its motion are determined, and a probable equation of the motion is proposed. It is demonstrated that there are two different kinds of motion of a bubble in water exactly analagous to the streamline and the turbulent flows of a viscous fluid; the motion passes from one kind to the other distinctly at the critical radius of the bubble.

(24)   Ohnesorge, W. V. "Die Bildung von Tropfen und die Auflösung flüssiger Strahlen," (Formation of Drops in Nozzles and the Dissolution of Liquid Jets). ZEITSCHRIFT FÜR ANGEWANDTE MATHEMATIK UND MECHANIK, Vol. 16, pp. 355-58. 1936.

By applying both Weber's surface tension criterion and Reynolds' viscosity criterion to liquid jets, the dimensionless quantity $Z = \mu \, (\sigma, \rho, d)^{-\frac{1}{2}}$ is presented (μ = absolute viscosity, σ = surface tension, ρ = density, and d = diameter of orifice.) When Z is plotted against the appropriate Reynolds number Re, the points on the resulting diagram, which are associated with jets of liquid shown by high-speed kinematography to possess screw symmetry, are confined between two parallel inclined lines. The band between these lines separates the points associated with streams showing plain axial symmetry from the points associated with streams showing spray formation. Re and Z are small for plain axial symmetry and large for spray formation.

(25)   Owens, J. S. "Experiments on Air-Lift Pumping," ENGINEERING, Vol. 112, pp. 458-61, September 1921.

The author reports on experimental investigations for the determination of the following:

1.   Relation of diameter of air bubble to velocity of rise through water.

2.   Effect of surface tension.

3.   Effect of diameter of orifice delivering air, and rate of flow of air on the size of the bubbles.

4.   Rate of oscillation of the bubbles.

The diameters of the bubbles were measured by means of a perforated plate and a wire loop. The observed results are:

1. The curve of velocity versus diameter bends rapidly downward towards a maximum as the bubble increases in diameter; it is stated that this is due to the flattening of the bubble.

2. The effect of nozzle diameter is very slight. The factor governing the size of bubble is the rate of air flow.

3. Surface tension, in the form of a film over the orifice, causes pulsation of air flow.

4. All bubbles oscillate rapidly as a result of the shedding of eddies.

(26) Peele, R. "Air Compression by the Direct Action of Falling Water." COMPRESSED AIR PLANTS, Chapter XIV. 1930.

This chapter presents a short summary on the theory of air compression by falling water and describes some of the existing installations where this principle is utilized.

(27) Pekeris, C. L. THE RATE OF RISE AND DIFFUSION OF AIR BUBBLES IN WATER, Office of Scientific Research and Development, No. 976, 1942. 29 pages.

A review of the results obtained by several investigators of relevant problems to the general study of the rate of rise and diffusion of air bubbles in water is presented in this article. A curve constructed using the data of previous investigators presents the rate of rise of air bubbles in water as a function of the radius of the bubble. For radii less than 0.04 cm, the bubble movement is like the movement of solid spheres. According to the author, this is due to the existence of a surface viscosity.

The rate of solution of bubbles depends upon whether they are stationary or moving. In the case of stationary bubbles, the experiments of Mache show that for large bubbles the rate of decrease of the square of the radius is constant, in accordance with the Mache-Epstein theory. Smaller bubbles dissolve at a less rapid rate and it is found that in

this case the interpretation of Mache's data can be improved by assuming that there exists a specific resistance to diffusion at the surface of the bubble. The magnitude of this surface resistance to diffusion has been approximately determined from Mache's data, and its effect on the lifetime of stationary bubbles is shown in a figure. It is seen that the effect is considerable, amounting to a doubling of lifetime in an extensive range of bubble radii.

In the case of moving bubbles it has been found experimentally that the radius decreases uniformly with time. The available experimental evidence on the rate of decrease of the radius is collected in a table. It is seen that even minute velocities of the order of 0.01 cm per sec markedly increase the rate of diffusion. The last calculation in the study presents the theoretical maximum rate of solution which results from the existence of a surface resistance to diffusion. The data are too meager to allow a determination of a solution-time-radius-curve for moving bubbles.

(28) Powell, Ralph W. "Vedernikov's Criterion for Ultra-Rapid Flow." TRANS-
ACTIONS OF THE AMERICAN GEOPHYSICAL UNION, Vol. 29, No. 6, pp. 882-86.
December 1948.

A criterion is given for the stability of steady uniform flow in open channels. When the number representing the criterion exceeds one, the flow is called ultra-rapid, roll waves form, and the flow cannot be steady. This criterion is compared with several which have been proposed, and is judged to be more comprehensive and exact by the author.

Data published previously by the author and reprinted in abbreviated form in this report were used to obtain an empirical formula for Chezy's "C" that would be valid in ultra-rapid flow. The assumption that the proposed criterion would be included in the law of resistance was checked by a statistical study of the data, but the best fit was obtained by including an additional term containing the Froude number.

(29) Robertson, J. M. and Rouse, H. "On the Four Regimes of Open Channel Flow." CIVIL ENGINEERING, Vol. 11. pp. 169-71. 1941.

The classification of flow into the four different regimes tentatively labeled (1) tranquil-laminar, (2) rapid-laminar, (3) tranquil-turbulent, and (4) rapid-turbulent is verified by analysis and experiment. Boundaries of the four regimes are given in a depth-velocity diagram. A brief discussion of the unsteadiness of rapid flow is presented with reference to the pulsating or slug flow found in steeper gradient channels that can exist over either the laminar or turbulent condition in the main flow stream.

(30) Schmidt, Ernst. "Uber die Bewegungsverhaltnisse von Gas-Flussigkeits-gemischen." (Flow Characteristics of Gas-Liquid Mixtures). V.D.I. 72, Hauptversammlung Trier, 1934.

Similitude considerations are applied to problems including the state of motion of an air bubble rising in an extended fluid and, accordingly, the characteristic magnitudes and the dimensionless criteria determining the process are established. By judicious combination the dimensionless criteria that would define the motion in this particular case are limited to two developed variables which may be defined in a single function. The Reynolds number of the bubble is expressed as a function of these dimensionless criteria. Results of experimental determination of this function can then be applied to computation of the velocity of rise of bubbles in any size in liquids of any given properties.

(31) Schnurmann, R. "Uber die Grösse von Gasblasen in Flüssigkeiten," (On the Size of Gas Bubbles in Liquids). Kolloid Zeitschrift, ZEITSCHRIFT FÜR WISSENSCHAFTLICHE UND TECHNISCHE KOLLOIDCHEMIE, Vol. 80, No. 2. August 1937. The David W. Taylor Model Basin, Translation 111, April 1943. 6 pages.

By taking a water-alcohol mixture as an example, it is found that the macroscopic sizes of the gas bubbles pressed through a filter are primarily a function of the viscosity of the liquid and not of its surface tension. A minimum bubble size corresponds to a maximum on the viscosity curve.

A curve showing the relation between velocity and size of bubble is plotted for the range of viscosity from 0.008 to 0.04 cgs. The curve shows that the bubble size increases inversely with the viscosity.

Experiments by Prausnitz and Halberstadt are discussed. The part played by the mechanism of coalescence of the subvisible gas bubbles previously discovered by the author is shown.

The pore diameter of the filter does not affect the rough qualitative effect, as long as the points of origin of the subvisible bubbles are close enough together to permit their collision.

(32) Scobey, F. C. THE FLOW OF WATER IN FLUMES. Department of Agriculture, Technical Bulletin No. 393. 1933. 98 pages.

Experiments on flumes were conducted for the determination of the coefficient of flow in Chezy's formula, the retardation factors in Kutter's and Manning's formulas, and the energy slope. Concrete, metal and wood flumes of various cross sections, sizes and lengths were used. Velocities ranged from 1.08 to about 30 fps. Interior conditions comprised surfaces new and old, clean and algae-coated, painted and unpainted, smooth and rough. Computations of $\underline{n}$ are made on the basis of $\underline{Q}$ measured by current meter, velocity measured by color method, and section determined from measurements by point hook gages and level rods, as well as on the basis of measured section and velocity determined as Q/A.

Results are compiled in tables. Recommendations are made for use of proper $\underline{n}$ in design, accounting for the phenomenon of air entrainment.

(33) Smetana, J. "Similitude des phénomènes hydrodynamique où l'eau est considérablement mélangée avec de l'air." (Similitude of Hydrodynamic Phenomena in which Water is Considerably Mixed with Air), REPORT ON FIRST MEETING OF THE INTERNATIONAL ASSOCIATION FOR HYDRAULIC STRUCTURES RESEARCH, Appendix 4, pp. 97-103. 1938. Translated by H. G. Dewey, U.S. Bureau of Reclamation, 1939.

An experimental investigation is discussed demonstrating the similitude of a hydraulic jump, both free and submerged, within the limits of scale from 1:1 to 1:10. The author finds no reason to doubt that similitude is valid for limits greater than the ones studied. The proof is based on a functional relationship established between the characteristics $F = V_1/\sqrt{gd_1}$, $\underline{p}$ of the jump, and the volume of air contained

in the roller of the jump. (V_1 and d_1 are the velocity and depth of the shooting flow and p is the degree of submergence of the jump.)

(34) Streeter, V. L. SECOND PROGRESS REPORT ON STUDIES OF THE FLOW OF WATER IN OPEN CHANNELS WITH HIGH GRADIENTS, U. S. Department of the Interior Bureau of Reclamation, Hydraulic Laboratory Report No. 40. October 13, 1938. 21 pages.

An evaluation of the data given in the first progress report on the same subject by C. W. Thomas [38] is presented. A design method for high gradient channels is proposed based on the following assumptions:

1. Air in and above the water causes an additional resistance which is directly proportional to the square of the average velocity and inversely proportional to some power of the hydraulic radius.

2. The hydraulic radius is based on a depth equal to the water discharge per foot of width divided by the average velocity in section, or the hydraulic radius assuming no air in the water.

3. The value of n in the Manning formula is constant for the particular type of material of which the chute or spillway is composed.

An additional term to account for air resistance is appended to the proposed equation of the flow and empirical coefficients for this term are calculated from the data of the Kittitas chute.

(35) Taylor, G. I. "The Viscosity of a Fluid Containing Small Drops of Another Fluid." PROCEEDINGS OF THE ROYAL SOCIETY, Series A, Vol. 138, pp. 41-48. 1932.

Einstein's expression for the viscosity of a fluid containing solid spheres in suspension is extended to include the case of liquid spheres. The expression obtained is valid provided the surface tension is great enough to keep the drops nearly spherical. When the rate of distortion of the fluid or the radius of the drop is great enough, the drops tend to break up and an approximate expression is given for determining the size of the largest drop that can exist in a fluid which is undergoing distortion at any given time.

(36) Thomas, C. W. PROGRESS REPORT ON STUDIES OF THE FLOW OF WATER IN OPEN CHANNELS WITH HIGH GRADIENTS. U. S. Department of the Interior Bureau of Reclamation, Hydraulic Laboratory Report No. 35, July 27, 1938. 28 pages.

A summary of experimental, analytical and field measurement studies of the Bureau of Reclamation on the subject of the hydraulic design of steep chutes and overfall spillways where air entrainment is an important factor is included in this report. A presentation of the particular problems facing the designer of these structures is accompanied by a discussion of the published results of several investigators of these same problems.

A detailed report of the tests made in a rectangular concrete chute 8 ft wide and 1300 ft long with a total drop of 340 ft is included. Slopes of various reaches of the chute ranged from nearly 0° up to 33° with horizontal, and discharges from 89 to 1005 cfs were recorded. Electrodes were installed in the channel at various stations and velocities were recorded by means of an oscillograph which indicated the location of a salt solution placed in the flow at the upstream end of the flume at the beginning of each run. Some difficulty was experienced in reading the oscillograph with entrained flow and the accuracy of the measurements could not be predicted. No consistent relationship among the variables involved was proposed after the analysis of the data.

Visual observation of the surface phenomena was abetted by the use of film records taken with a camera at 64 frames per second and then projected at 16 frames per second. Verification of the existence of water slugs traveling over the main water flow with a greater velocity than the underlying mass was made with this photographic technique.

(37) Thomas, H. A. "The Propagation of Waves in Steep Prismatic Conduits." PROCEEDINGS OF HYDRAULICS CONFERENCE, University of Iowa, Bulletin 20, pp. 214-29. March 1940.

A study of channels of extreme steepness, where a train of waves or pulses distinguishes the flow, is reported in this paper. The author restates the proposal that a necessary condition for the existence of this flow is a slope four times as great as the ordinary critical slope between streaming and shooting flow. Moving belt equipment and analogies were used in preference to a purely mathematical analysis in order to lend an element of tangibility to the study of the traveling-wave problem.

(38) Uren, L. C., Gregory, P. O., Hancock, R. A. and Feskov, G. "Flow Resistance of Gas-Oil Mixtures Through Vertical Pipes." TRANSACTIONS OF THE AMERICAN INSTITUTE OF MINING ENGINEERS, Petroleum Division, Vol. 86, pp. 209-19. 1930.

Experimental investigations for the determination of the flow characteristics of gas-oil mixtures under certain conditions are reported on by the authors. The results indicate the following:

1. The density and viscosity of gas-oil mixtures vary as logarithmic functions of the gas-oil ratio.

2. Within the range of gas-oil mixtures commonly produced from flowing and gas-lift wells, an increase in the volume ratio of gas to oil, at a given pressure, results in increased pressure loss per linear unit of flow into the eduction tube.

3. Increase in pressure for a given gas-oil ratio or a given flow velocity is productive of decreased unit pressure loss. Therefore, a given gas-oil mixture is moved with lower unit pressure loss through the lower part of an eduction tube where the pressure is high, than through the upper part where the pressure is comparatively low.

4. A critical gas-oil ratio probably exists for any set of conditions at which the pressure loss in flow through tubing is a minimum, but this most efficient ratio is probably lower than gas-oil ratios realized in well operation.

5. In applying the Fanning equation to the flow of gas-oil mixtures through vertical pipes, the friction factor f is determinable by a proposed logarithmic function.

(39) Vedernikov, V. V. "Characteristic Features of a Liquid Flow in an Open Channel." COMPTES RENDUS (DOKLADY) DE L'ACADEMIE DES SCIENCES DE L'URSS, Vol. LII, No. 3, pp. 207-10. 1946.

In this paper the applications of the author's criterion for the establishment of ultra-rapid flow in motions that are initially uniform and steady are discussed. A consideration of the criterion for motions that are initially steady, but nonuniform, is added along with references to laminar motions that are found to be ultra rapid. The author suggests a modified form of the criterion (based in the region where the movement is nearly uniform) that would be useful in determining the reach necessary for the establishment of ultra-rapid flow.

(40) Vedernikov, V. V. "Conditions at the Front of a Translation Wave Disturbing a Steady Motion of a Real Fluid." COMPTES RENDUS (DOKLADY) DE L'ACADEMIE DES SCIENCES DE L'URSS, Vol. XLVIII, No. 4, pp. 239-42. 1945.

A mathematical analysis of the conditions at the front of a translation wave disturbing a steady motion of a real fluid is presented by the author. A criterion for the onset of ultra-rapid flow in an open channel is presented.

(41) Versluys, J. "The Cause of Periodicity Generally Occurring with Rising Mixtures of Gas and Liquid." PROCEEDINGS, KONINKLIJKE AKADEMIE VAN WETTENSCHAPPEN TE AMSTERDAM, Vol. 33, No. 1-5, pp. 450-56. 1930.

The phenomenon of periodicity of flow of mixtures of gas and liquid rising in a vertical channel is explained on the basis of the assumption that two conditions are possible in the mixture, the foam condition and the mist condition. Which of these conditions will arise depends upon the proportion in which gas and liquid are mixed. This proportion is determined not only by the proportion in which they flow out, but also by their difference of speed and their absolute velocity. Due to the fact that difference of speed is not the same under both conditions, direct transition from one to the other is impossible. If intermediate conditions prevail in any part of the channel, intermittent flow will be the result.

(42) Wilde, H. D. and Moore, T. V. "Experimental Measurement of Slippage in Flow Through Vertical Pipes." TRANSACTIONS OF THE AMERICAN INSTITUTE OF MINING ENGINEERS, Vol. 92, pp. 296-319. 1931.

This paper presents results of some work conducted for the purpose of measuring slippage in short experimental gas lifts. It is shown that slippage in a vertical pipe carrying a mixture of oil and gas can be easily calculated if the fractions of the pipe occupied by liquid and gas, respectively, are known. In this work, the fraction of the pipe occupied by the liquid was measured under a wide variety of conditions and the relationship between this fraction and other quantities, which are easily measured in ordinary work was determined.

The purely empirical correlation of the data presented in the paper is valuable because it shows the qualitative relationship between the variables involved. For a given rate of flow of liquid and gas, slip losses are less in smaller pipes and friction losses are greater. The most efficient flow string is one that balances the slip and friction losses so that a given quantity of oil and gas can be carried with a minimum pressure drop. Slippage is not affected by the viscosity of the flowing liquid. It is, however, dependent markedly upon the density and to a large extent upon the surface tension of the liquid. The quantitative relationship given can be used satisfactorily in computations for short lifts.