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STUDIES OF AIR ENTRAINMENT IN OPEN-CHANNEL FLOWS

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Synopsis

The phenomenon of insufflation of air into water flowing at high velocities in open channels has long been of great interest to hydraulic engineers, but detailed experimental information regarding the occurrence and the investigation of the flow mechanism of air-water mixtures has been elusive because of an inadequacy of accurate observations made by means of customary instrumentation. New instruments for measuring velocities of mixed flows and the air concentration in such flows have been devised at the St. Anthony Falls Hydraulic Laboratory of the University of Minnesota, at Minneapolis, so that velocity traverses and air-entrainment traverses can now be made accurately for widely diverse flow conditions, both for velocities and percentages of air entrained.

This paper describes the results of experimental observations of self-aerated flows in an open channel for various slopes of from 15° to 45°. It is pointed out that customary open-channel flow relationships do not apply directly to air-entrained flows. For the range of conditions reported, actual velocities are shown to be greater than the velocities computed by hitherto proposed methods. The experiments reported are for the conditions of a smooth, painted-steel channel surface. The authors contemplate further studies with channels of differing roughness and greater slopes.

Introduction

The experimental investigation of self-aerated flows in flumes with steep gradients is a phase of the air-water mixture studies now in progress at the St. Anthony Falls (SAF) Hydraulic Laboratory of the University of Minnesota, at Minneapolis. These experiments are being conducted in a large laboratory facility (Fig. 1) especially designed and constructed to provide naturally

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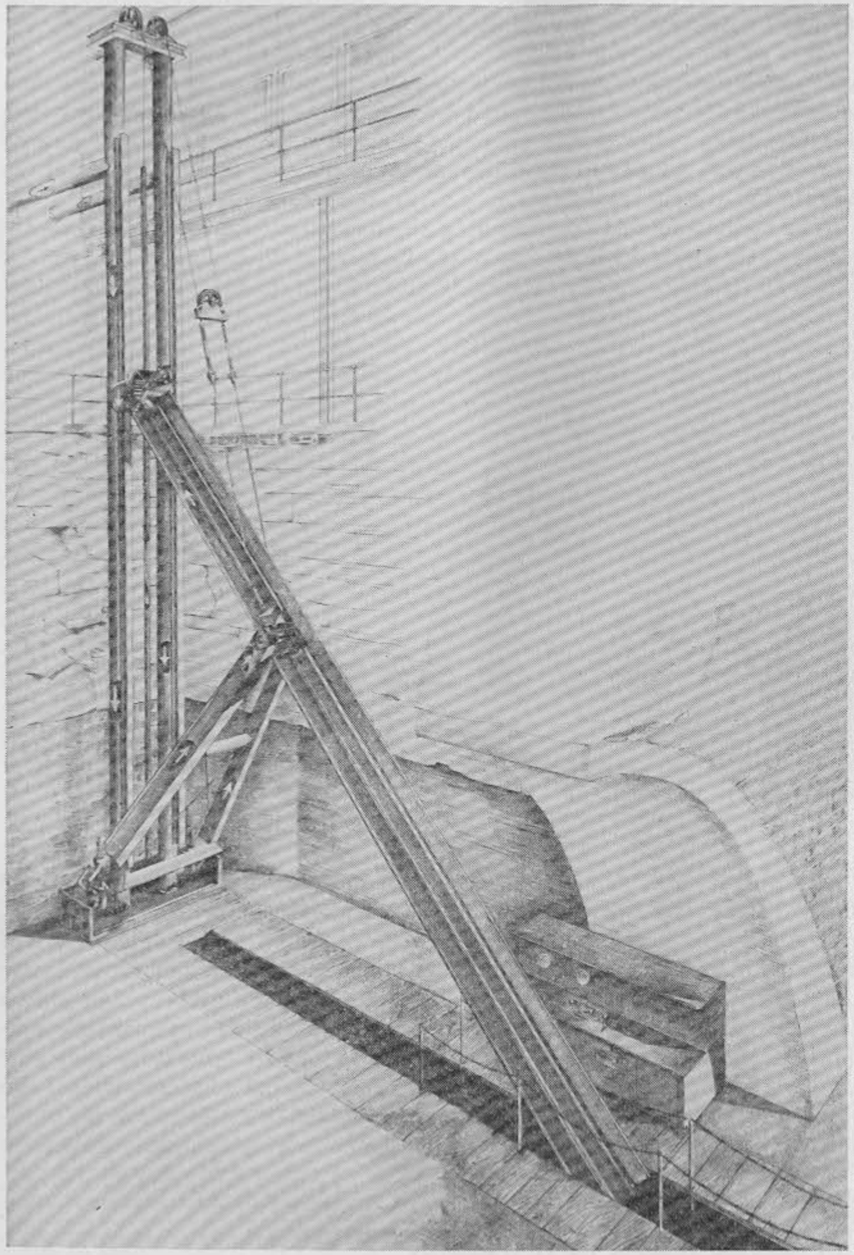

Fig. 1.—Channel for Air-Entrainment Study at the St. Anthony Falls Hydraulic Laboratory

aerated flows through a considerable range of discharges and at all flume slopes ranging from horizontal to vertical.

It was immediately evident that the customary instrumentation for fluid-flow observations would be inadequate; it would supply no more than superficial information which might be misinterpreted. Much attention was therefore given to devising instruments to measure the local values of air concentration and velocity in small increments of the cross-sectional areas of the flows in the experimental flume.

The method devised for measuring air concentration involves the determination of the electrical resistance of the filament of fluid mixture bounded by two small electrodes. The Maxwell equation for the resistance of a suspension of spheres in a conducting medium was utilized to initiate the design. An equivalent electric circuit was developed to adapt the expression to air-entrainment conditions in which the fluid mixture does not always occur as a homogeneous combination of the air phase and the water phase. The instrument, as it is now being used, is accurate, direct-reading, easily handled, and does not require calibration by volumetric means for its operation. The design and development of the apparatus are described in a separate publication.[3]

The method of measuring velocity in small filaments of the two-phase flows of air and water has been described in a previous publication.[4] Being a basic length/time determination over a very short distance (3 in. to 4 in.), the method required precise injection and timing components. Timing precision in the neighborhood of 1/100,000 sec and appropriate pickup and amplifying stages were incorporated into a unit electronic circuit; a diesel injection mechanism was used to achieve the necessary precise identification of the measured filament. The instrument is uniquely fitted for air-entrainment studies as it depends neither on a knowledge of the fluid density nor on a strict homogeneity of the fluid or mixture.

Description of the Flow

R. Ehrenberger, in his pioneering work on open-channel aerated flows,[5] loosely divided the flow into several zones: Water near the floor of the flume, individual air bubbles in water, a mixture of water and air, individual drops of water in air, and an overlying movement of air. Several writers subsequently adopted this arbitrary classification as a real delineation between flow phases and attributed special qualities to the intermediate phases to explain observations they had made. Earlier measurements at the SAF laboratory using flow sampling methods to obtain air concentrations[6] showed conclusively that the mean air concentration increased continuously with elevation above the flume bottom. The mean air concentration at the point of measurement closest to

[3] "An Electrical Method for Measuring Air Concentration in Flowing Air-Water Mixtures," by Owen P. Lamb and J. M. Killen, *Technical Paper No. 2*, Series B, St. Anthony Falls Hydr. Lab., Univ. of Minnesota, Minneapolis, Minn., March, 1950.

[4] "Velocity Measurements of Air-Water Mixtures," by Lorenz G. Straub, J. M. Killen, and Owen P. Lamb, *Proceedings-Separate No. 193*, ASCE, May, 1953.

[5] "Wasserbewegung in steilen Rinnen (Schusstennen) mit besonder Berucksichtigung der Selbstbelüftung," by R. Ehrenberger, *Österreichischer Ingenieur und Architektverein*, Nos. 15/16 and 17/18, 1926.

[6] "The High Velocity Flow of Water in a Small Rectangular Channel," by W. W. DeLapp, thesis presented in 1947 to the University of Minnesota, at Minneapolis, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

the flume floor varied considerably between flows, depending on roughness of the bottom, flow depth, mean flow velocity, and the distance from the initiation of aeration. Whatever the value of this lowest measured point, there was established a definite gradient of air concentration with no indication of layer flows or sharply separated flow phases.

The hypothesis that aeration becomes incipient in a flume or on a spillway when the turbulent boundary layer from the channel bottom intersects the water surface was forwarded by Emory W. Lane,[7] M. ASCE. Later and more elaborate discussions[8,9] substantiated his hypothesis, showing close agreement between computations and observations in several cases.

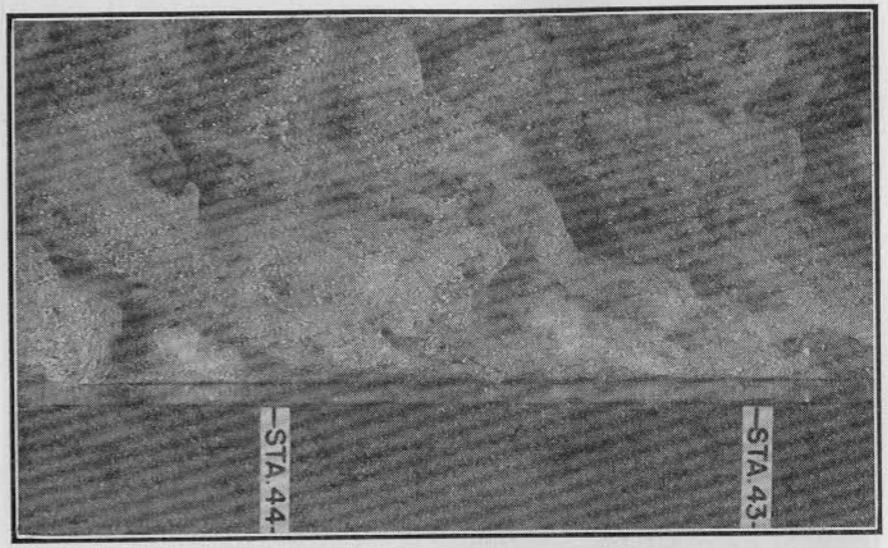

Fig. 2.—Surface of Fully Aerated Flow (Exposure: 1/100,000 Sec)

A characteristic roughening of the water surface immediately before the appearance of "white water" can readily be seen in most free-surface installations where self-aeration occurs. Further observations with high-speed photographs such as Fig. 2 illustrate the persistence and magnification of this roughening of the interface into and during the highly aerated condition so that at any instant the aerated flow has the appearance of exceedingly rough and irregular topography. The depressions or open pockets of air at the interface above the dense underlying fluid and the spikes of water mixture projecting into the air are of varying dimensions and are spaced in seemingly random occurrence. This violent agitation of the interface between the mixture and the air causes quantities of air to be entrapped and broken into bubbles of a size that presents a balance between the work of agitation of the liquid and the

[7] "Entrainment of Air in Swiftly Flowing Water," by Emory W. Lane, *Civil Engineering*, February, 1939, pp. 89–91.

[8] "Air Entrainment on Spillway Faces," by G. H. Hickox, *ibid.*, December, 1945, pp. 562–563.

[9] "Study of Entrainment of Air by Flowing Water," by G. Halbronn, thesis presented in 1951 to the University of Grenoble, France, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

surface energy of the resultant bubbles. Thus, the air bubbles in water are approximately spherical in shape and nearly the same general size for a particular set of flow conditions. This fact is illustrated in Fig. 3, which is a view through a transparent side wall into a partly aerated flow.

FIG. 3.—PROFILE OF PARTLY AERATED FLOW (EXPOSURE: 1/50,000 SEC)

Transport of bubbles farther into the flow by turbulence is counteracted by the buoyant force on each bubble so that a balanced condition is reached wherein as much air is buoyed out of the flow as is drawn into it. This insufflation balance can be compared to the suspended-load balance in a reach of a sediment-

carrying stream in which neither aggradation not degradation occurs. When the flow has enveloped as many air bubbles as it can retain and when the rough surface topographies at adjoining sections along the channel have become statistically similar, the flow can be defined as having reached a condition of normal aeration for the particular section, slope, roughness, and discharge.

A portion of the discharge of water through a section in aerated flow consists of water droplets that have become detached from the stream and move through the air by virtue of their initial momentum. The force of gravity pulls them downward and they soon fall back into the flowing stream, with the result that, on the average, only a small percentage of the total water discharge flows as droplets at any slopes appreciably less than 90° with the horizontal. Air bubbles in the water tend to follow the turbulent fluctuations of the flow and to have the same mean velocity as the water enveloping them. There is not likely to be any appreciable "slip" in either the upstream or the downstream direction of these isolated bubbles in open-channel flows as compared to the bulk-flow velocities. Water droplets moving through the air, however, would have an appreciable velocity relative to the air, which tends to shorten their trajectories and, consequently, the time intervals spent as isolated droplets.

The roughened surface and the high flow velocities of the flow mixture tend to drag a considerable volume of air into motion above the flow. A velocity deficiency can be expected in the air-water mixture in the region of the roughened surface because of the drag necessary to sustain this air motion.

At relatively small flow depths, flows on steep gradients break into waves which appear to roll down the channel over a thin underlying stream of water. Bubbles are often entrapped into these wave fronts, and the "white water" has the appearance of regularly spaced bands or wave patterns moving down the slope. At slightly greater depths where a more general condition of aeration prevails, vestiges of these periodic waves can often be detected by their regularity as slightly higher stages immediately followed by pockets of greater air content. Formation of "roll waves"[10] without consideration of aeration is discussed in a recent summary of the problem which refers to preceding papers on the same subject.

The variety of differing and, in many cases, opposing opinions expressed by investigators of air-entrainment phenomena encountered in a survey of the literature[11] illustrates the many uncertainties that exist in this field. Distributions of density, of velocity, of water discharge, and of momentum flux in a cross section of aerated flows are of interest both for design purposes and ultimately for an understanding of the flows. Bulk increase of section by aeration, comparison of mean velocities with velocities in nonaerated flows, and the possibility of aeration are vital design considerations.

EXPERIMENTAL OBSERVATIONS

Experimental measurements described in this paper are largely restricted to velocity and air-concentration traverses in flows where normal aeration has been attained. Air concentration C is defined as the volume of air in a unit volume

[10] "Resistance Effects on Hydraulic Instability," by R. F. Dressler and F. V. Pohle, *Communications on Pure and Applied Mathematics*, Vol. VI, 1953, pp. 93–96.

[11] "Air Entrainment in Flowing Water," by Owen P. Lamb, *Project Report No. 19*, St. Anthony Falls Hydr. Lab., Univ. of Minnesota, Minneapolis, Minn., August, 1949.

of air-water mixture. The concept of mixture is intended to embrace both the phase in which air bubbles are in the water and the condition where the unevenness of the surface contributes to the variation of density in planes parallel to the flume floor. The component directions x and y are taken positive in the downstream direction of the flow and the upward normal to the flume floor, respectively. The upper boundary of the flow has been arbitrarily defined as that value of y at which $C = 0.95$. This boundary envelops approximately from 98% to 99% of the total water discharge and can still be reliably measured with the concentration instrument. The velocity in the positive x-direction

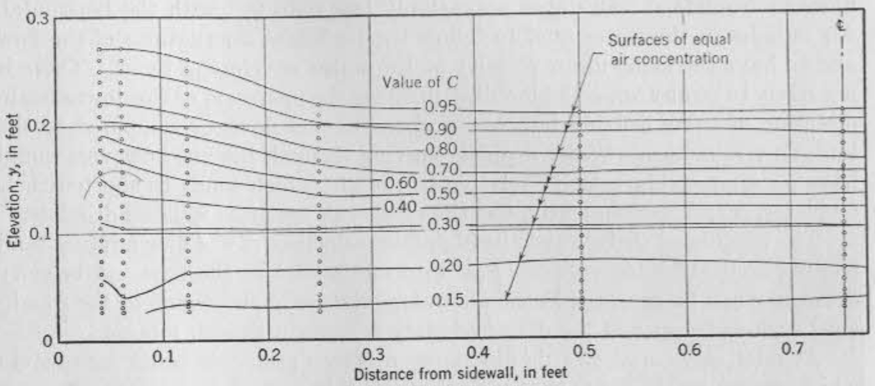

Fig. 4.—Air Concentration Distribution Across Section of Fully Aerated Flow

is labeled u and is the only velocity component measured with the SAF velocity meter. Direct velocity measurements can be obtained up to an elevation y that is approximately $\frac{1}{2}$ in. smaller than $\bar{y}$.

The summation $\sum_{y=0}^{y=\bar{y}} (1 - C)\Delta y$ is the total area occupied by water in a unit breadth of the flow cross section.

The summation $\sum_{y=0}^{y=\bar{y}} u(1 - C)\Delta y$ is the total water discharge q_w in a unit breadth of the flow cross section.

The mean velocity, $\bar{U}$, of the water through the section is obtained by dividing the second summation by the first—that is, by directly applying the equation of continuity to the measurements. The mean air concentration, $\bar{C}$, is equal to the summation $\sum_{y=0}^{y=\bar{y}} C\,\Delta y$ divided by $\bar{y}$ and is seen to depend in absolute value on the choice or definition of $\bar{y}$.

The total water discharge, Q'_w, in the 1.5-ft-wide flume is measured at metering stations in the supply pipes leading to the equipment. Discharge per unit width $q'_w = \frac{2}{3} Q'_w$ is used to compare with the integrated water discharge obtained with the instruments in the aerated flows and to act as a control on those measurements.

The inlet of the test flume is designed so as to present a uniform velocity profile to the jetted inflow at a controlled depth. Inlet velocity is adjusted until it is approximately the terminal velocity for the set slope and discharge. Thus, the flow aeration can be studied independently of large accelerative or decelerative effects, and a shorter flume with a greater flexibility and control can be utilized. Orientation of flume components was selected to provide identical inlet geometry throughout the 90° range of flume slopes. Until the present time (September, 1953), the equipment has been operated at flume slopes up to 45° with the natural gravity head of 50 ft available at the site, but with the

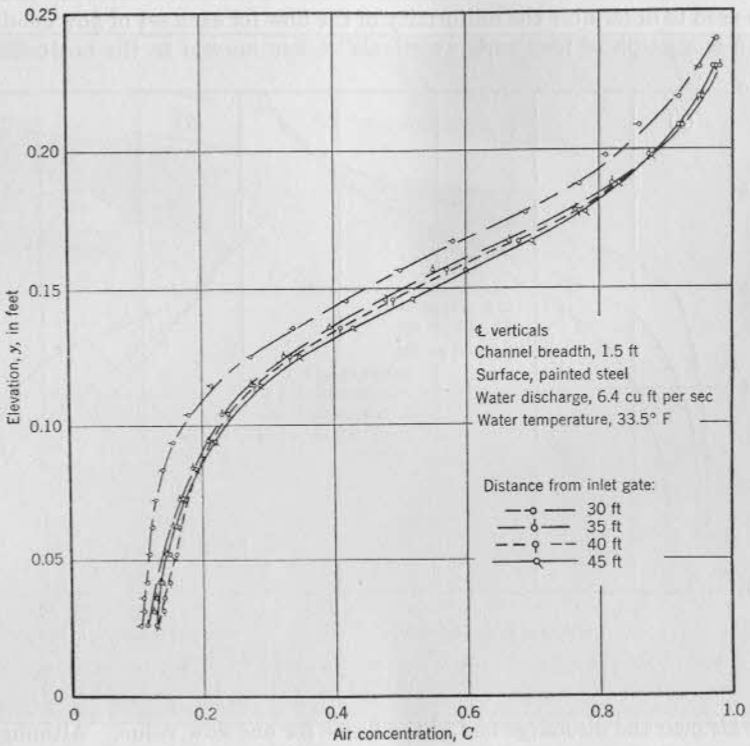

Fig. 5.—Measured Air-Concentration Values at Various Distances from the Flume Inlet Gate

completion of a pumped-supply system currently being installed, flows with jetted terminal velocities at flume angles approching 90° will also be tested.

The 1.5-ft breadth of flume is sufficiently large compared to the flow depths used to preserve a two-dimensional region of the flow several inches wide and symmetrical about the flume center line. Complete traverses across the entire cross section of the flume reveal no asymmetry about the center line for either nonaerated or aerated flow. Interpolated lines of equal air concentration drawn between measured values in normal aerated flow at a station 45 ft from the flume inlet are exhibited in Fig. 4. The 0.95 level of air concentration is considerably higher near the side wall than in the central region of the stream. This side-wall effect is especially noticeable before aeration begins in the central region; aeration at the sides is initiated by the turbulence from the side-wall boundary at the water surface and thus appears as "white water" sooner. The flat contours in the larger central region of the stream (Fig. 4) illustrate the two-dimensional character of that flow which remains uninfluenced by side-wall disturbances and is typical of the center-line region for all flows for which complete cross-sectional traverses were obtained. The remaining data presented in this paper were obtained in center-line vertical traverses of the stream.

Air-concentration vertical traverses at various distances from the inlet gate were used to determine the uniformity of the flow for each set of flow conditions. Fig. 5 is a graph of four such "verticals" taken normal to the bottom at 5-ft

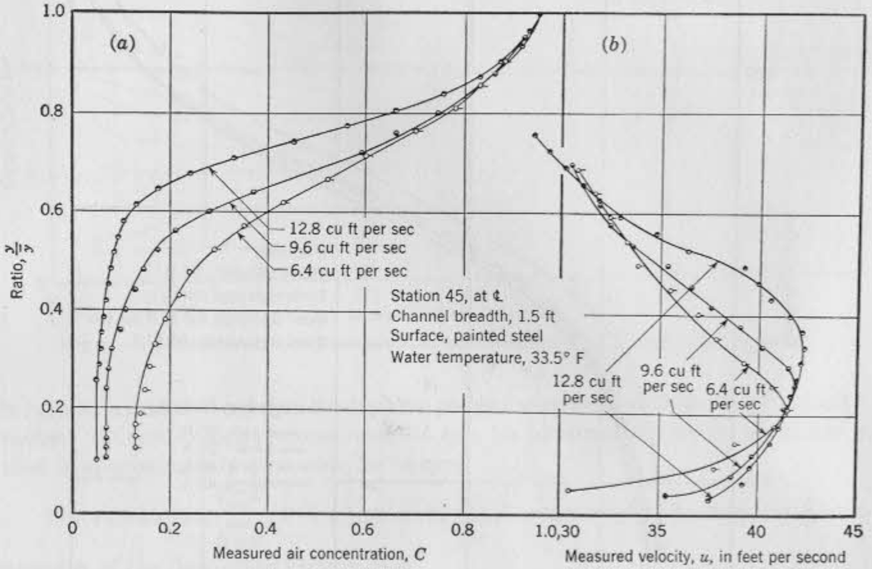

FIG. 6.—DISCHARGE COMPARISON, AERATED FLOWS AT VARIOUS DISCHARGES WITH A 15° FLUME ANGLE

intervals near the discharge end of the flume for one flow value. Although the profiles for stations located 35 ft, 40 ft, and 45 ft from the inlet gate exhibit small differences, they are definitely closely grouped and, considering the measuring variation in the positioning and reading of the instrument, they cannot be considered as significantly different. On the other hand, the profile for station 30 is definitely different from the other three to an extent that is typical of changes occurring in the same increment of flume length either in flows with larger dicharges or at stations even closer to the inlet at which full normal aeration has not been established. For the test data here reported, a condition of normal aeration was substantially reached at station 45 but not at station 30. The remaining data presented in this paper were taken in the center-line "vertical" of the flows 45 ft from the inlet gate and are classified as fully aerated or partly aerated depending on their similarity to upstream profiles.

Water discharges of 6.4 cu ft per sec, 9.6 cu ft per sec, and 12.8 cu ft per sec in the 1.5-ft-wide flume inclined 15° can be compared by means of the air-concentration profiles in Fig. 6(*a*) and the velocity profiles in Fig. 6(*b*). From Fig. 5 it was determined that the 6.4-cu-ft-per-sec flow had reached normal aeration, whereas respective data for the other flows indicated that the 9.6-cu-ft-per-sec flow was approaching normal aeration and that the 12.8-cu-ft-per-sec flow was definitely not normal but only partially aerated at station 45 where it had not yet reached a uniform or normal state.

Observations of these and many other flows with stroboscopic light and in some instances with high-speed photographs such as Fig. 7 indicate that the

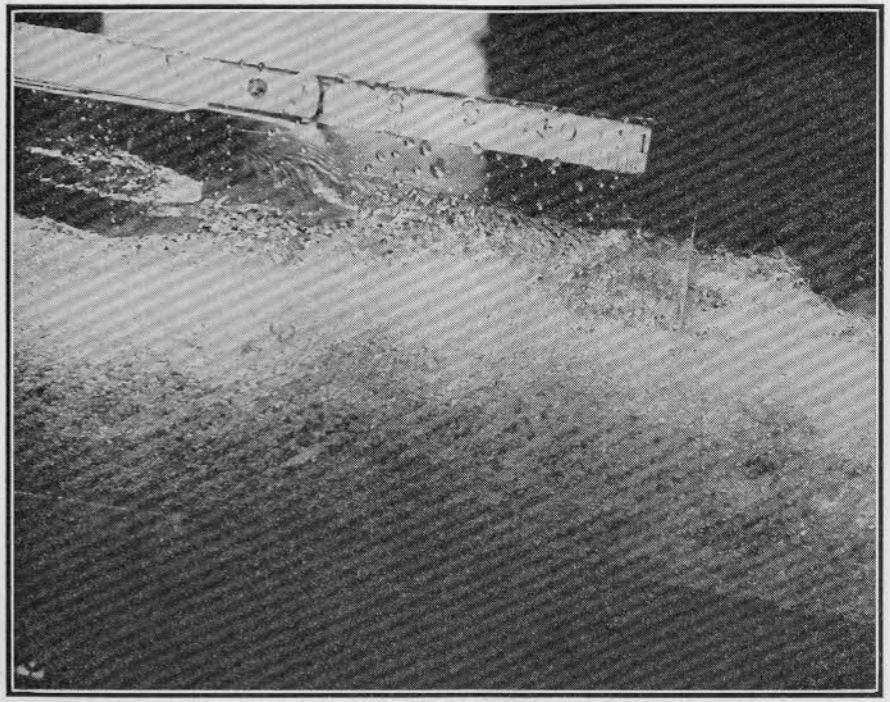

FIG. 7.—MEASURED VELOCITIES IN AERATED FLOWS AT VARIOUS DISCHARGES WITH A 15° FLUME ANGLE

rapid increase in air-concentration gradient begins at an elevation above the flume floor corresponding to the lowest depressions or pockets of air. The air concentration below this elevation is due to the air bubbles in the water, some of which migrate far down into the stream even at low aerations.

Several uniform nonaerated flows were set at lower flume angles to form a basis of comparison for the values of mean velocity and for the distributions of discharge and momentum flux obtained with the aerated flows. Typical of these is the condition shown in Fig. 8, a nonaerated flow on the same painted-steel surface of the 1.5-ft-wide test flume as the aerated flows described herein.

tion values from 0.39 at a 7.5° flume angle to 0.472 at a 45° flume angle is quite small and suggests that the bulking of the flows is not particularly sensitive to slope changes alone. Because the runs at varying discharges were not all established as reaching a condition of normal aeration, the respective mean air-concentration values are not comparable on the basis of different discharges. A comparison of the slope and discharge for the nonaerated run of Fig. 8 with the slope and discharge of the normal aerated run on the 7.5° flume angle shows very little difference between the flow variables; however, a bulking of the flow equivalent to a mean air concentration of 0.39 and considerably higher velocities were measured in the aerated flow. This suggests possible critical flow criteria for the conditions at which aeration begins and requires further experimental study.

The defined flow depth $\bar{y}$ was changed very little by a change of slope for the various runs at constant discharge. Within the narrow range of the reported experiments, the tendency to increase the section by greater bulking appeared to be practically compensated for by a further increase in velocity so that flow depth remained nearly constant.

The relations between slope and velocity in the uniform open-channel flow in most common usage are empirical modifications of the semirational Chezy equation. The Manning form of the equation is familiar to the engineering profession and has been widely accepted for design purposes in conventional mild-slope channels. However, doubts have persisted as to the validity of any of the conventional expressions in cases of channels of steep gradients or where high velocities can be expected. With aeration also a consideration, the problem of predicting mean velocity at steep gradients becomes far more complicated.

In earlier papers on the subject of air entrainment,[12,13] computed values of the hydraulic radius to be used with conventional n-values have been proposed. In one case, the computed values of the hydraulic radius are based on an area and the wetted perimeter of a computed shallower section occupied by water alone, and, in the other case, the computed value is based on the area of the shallower section of water alone and a wetted perimeter of the full-flow section. Using these expressions and the data presented in Table 1, n-values are obtained by such assumptions which show no agreement with the n-value for uniform flow in the flume with no aeration. For the latter situation corresponding to flow in Fig. 8, for example, the n-value was 0.0106.

Combining the Manning equation for a unit breadth of channel with the equation of continuity, an expression for velocity in terms of discharge, slope, and roughness value is obtained:

$$V = \left(\frac{1.486}{n}\right)^{0.6} q^{0.4}{}_w\, S^{0.3}.$$

Using the measured value of $q_w = 4.267$ from Table 1 and $n = 0.0106$ derived in Fig. 8, the following values of mean velocity are obtained for flume angles from 15° to 45°:

[12] "Open Channel Flow at High Velocities," by L. Standish Hall, in "Entrainment of Air in Flowing Water: A Symposium," *Transactions*, ASCE, Vol. 108, 1943, pp. 1394 and 1494.

[13] "Second Progress Report on Studies of the Flow of Water in Open Channels with High Gradients," by V. L. Streeter, *Hydraulic Laboratory Report No. 40*, Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C., October 13, 1938.

Flume slope, in degrees	Mean velocity, V (ft per sec), Manning formula	Measured mean velocity (from Table 1)
15	23.1	35.38
22.5	26	35.93
30	28.1	36.25
37.5	29.8	37.54
45	31.2	39.79

Thus, by comparing these computed mean velocities from the Manning formula with the measured $\bar{U}$-values from Table 1, it will be noted that in all cases the velocity computed by the assumption for unaerated flows is too low.

The importance of increase of section or of hydraulic radius because of aeration in explaining the high values of mean velocity must be minimized

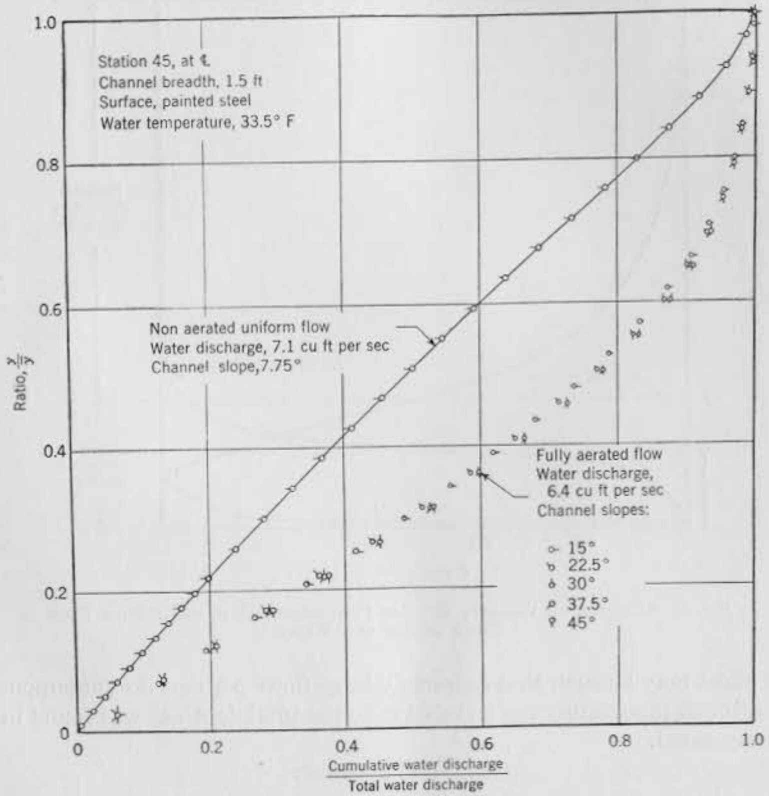

FIG. 10.—DIMENSIONLESS REPRESENTATION OF THE CUMULATION OF WATER DISCHARGE WITH ELEVATION ABOVE FLUME BOTTOM

because, as can be seen from the velocity profiles in Figs. 6(*b*) and 9(*b*), any increase in section is more than offset by the upper-surface drag on the overlying air, which causes a decrease in velocity to less than the mean velocity throughout the added area.

Fig. 10 illustrates the difference between cumulative water discharges in the flow section for fully aerated and nonaerated flows. It will be observed that, for the test conditions recorded, nearly 90% of the water discharge is in the lower 0.6 of the depth in the case of the aerated flow.

Fig. 11 is a generalized diagram showing the curves of velocity and air concentration in aerated flow, modeled from the curves that were obtained by direct measurement. The position of the arbitrarily defined flow surface with respect to the entire flow field is indicated by the dotted line. The portion of the velocity curve above the defined surface refers to the moving air mass and not to the scattered water droplets that would be expected to have velocities closer to the velocities of the lower flow regions from which they were ejected. The general shape of these curves is believed to be characteristic of fully aerated flows although details of the profiles may vary with depth, roughness, and slope.

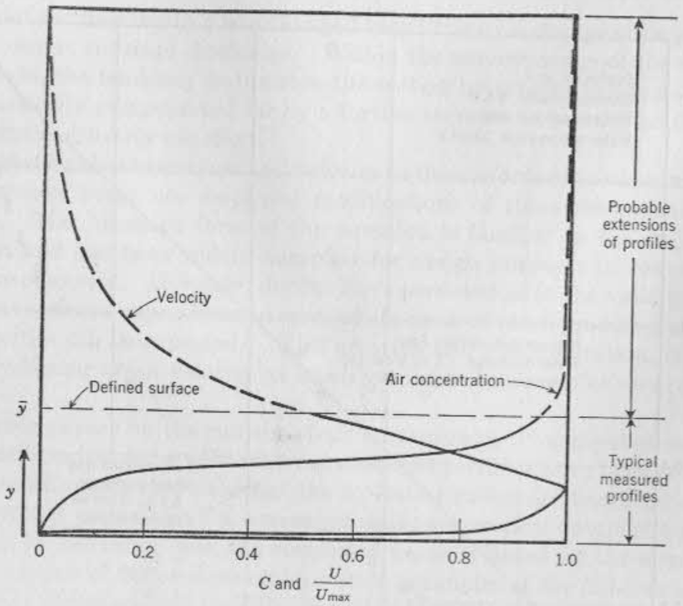

FIG. 11.—PROFILE OF VELOCITY AND AIR CONCENTRATION IN THE ENTIRE FLOW FIELD OF AIR AND WATER

Scale effect may be such that extremely large flows perhaps do not support as large a rough topography region relative to the total depth as was found in the flows measured.

ACKNOWLEDGMENT

Exploratory air-entrainment research studies at the St. Anthony Falls Hydraulic Laboratory were begun in 1939 under the sponsorship of the Engineering Foundation through the ASCE Committee on Hydraulic Research. Later and more comprehensive work has been undertaken in recent years under the sponsorship of the Office of Naval Research, United States Department of Defense.