

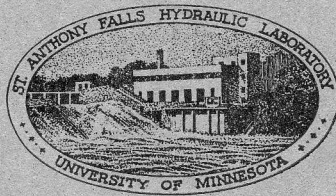
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UNIVERSITY OF MINNESOTA  
ST. ANTHONY FALLS HYDRAULIC LABORATORY  
LORENZ G. STRAUB, Director

Technical Paper No. 2, Series B

# An Electrical Method for Measuring Air Concentration in Flowing Air-Water Mixtures

by  
OWEN P. LAMB  
and  
JOHN M. KILLEN



March, 1950  
Minneapolis, Minnesota

A N E L E C T R I C A L M E T H O D F O R M E A S U R I N G  
A I R C O N C E N T R A T I O N I N  
F L O W I N G A I R - W A T E R M I X T U R E S

SUMMARY

An electrical instrument has been developed to measure the air concentration in flowing air-water mixtures as part of an investigation of the mechanism by which atmospheric air is entrained in flowing water. The theory, development, and verification of this instrument are described in this report. The instrument is shown in Figs. 6c and 7, and the electrical circuit is shown in Fig. 9b.

The electrical measurement of air concentration was chosen after examining possible mechanical, chemical, and magnetic methods. The method consists basically of a measurement of the difference between the conductivity of a mixture of air and water and conductivity of water alone. A mechanical strut supporting a pair of electrical probes has been combined with the electrical circuit in such a manner that air-concentration measurements may be made, not at a point, but at least in a small region of the flow. With this instrument it is now possible to traverse the flow cross section both vertically and laterally and to obtain the distribution of air in the flowing mixture. The relation between the instrument readings and air-concentration values are determined theoretically and the instrument is direct reading. By checking the results of the electrical method against a direct mechanical sampling method, it has been determined that the values of air concentration computed from theoretical conditions alone were sufficiently accurate for experimental work without further calibration.

The instrument is now designed for application in an experimental laboratory channel, but with the use of a more rugged strut and supporting structure the method could be applied equally well in large flumes and spillways.

I. INTRODUCTION

The "white water" commonly observed on steep flumes and spillways is important to the designer of these structures, but the information about it is inadequate for efficient design purposes. The white appearance of the water is caused by the entrainment of air in the water, a process known as

self-aeration. Self-aeration occurs at high flow velocities on slopes of  $6^{\circ}$  or more, and, if the slope is long enough, produces a maximum terminal air content in the water beyond some point on the slope. Several mutually exclusive theories have been used to define the self-aeration of the water and the resulting flows of air-water mixtures. Experimental observations have not been sufficiently precise to exclude the impractical theories and to act as a guide for the development of the correct analysis. This dearth of reliable experimental data can be attributed to the inability to obtain accurate distributions of velocity and air concentration in the flow cross section with available instruments.

For the above reason, it was decided to concentrate on the development of satisfactory instruments for the measurement of the air-entrainment phenomena before a comprehensive experimental program was launched. A variable-slope, laboratory flume (Figs. 1 and 2) and a mechanical, air-water sampling device (Figs. 3 and 4), which had been used in previous experimental programs on air entrainment at the St. Anthony Falls Hydraulic Laboratory, were available for use in instrument development. Unpublished data obtained with the above equipment, as well as previously published reports [6]\*, were used to establish the requirements of the instruments to be developed.

The nature of the admixture of air and water is of greatest importance to the development of specialized instruments. Photographic observations, tempered by basic analyses of the physical state of the mixture, form the basis for the following description of the flow. Air bubbles are surrounded by the water up to a level of mean air concentration between 50 and 60 per cent. The majority of these bubbles adhere closely to the spherical shape, because when they are large enough to become appreciably distended, the forces of motion tend to cause a rupture of the bubble and a consequent division into spheroids of smaller size. Above the bulk of the water flow, a number of water droplets are in motion through the air; these droplets make up a very small percentage of the total water flow through the section. At the levels of air concentration from 60 to 95 per cent, there is an extremely irregular traveling-surface region. This surface is undergoing constant change normal to the direction of the bulk flow, but has no apparent velocity in the

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\*Numbers in brackets refer to the corresponding numbers in the Bibliography, p. 28.

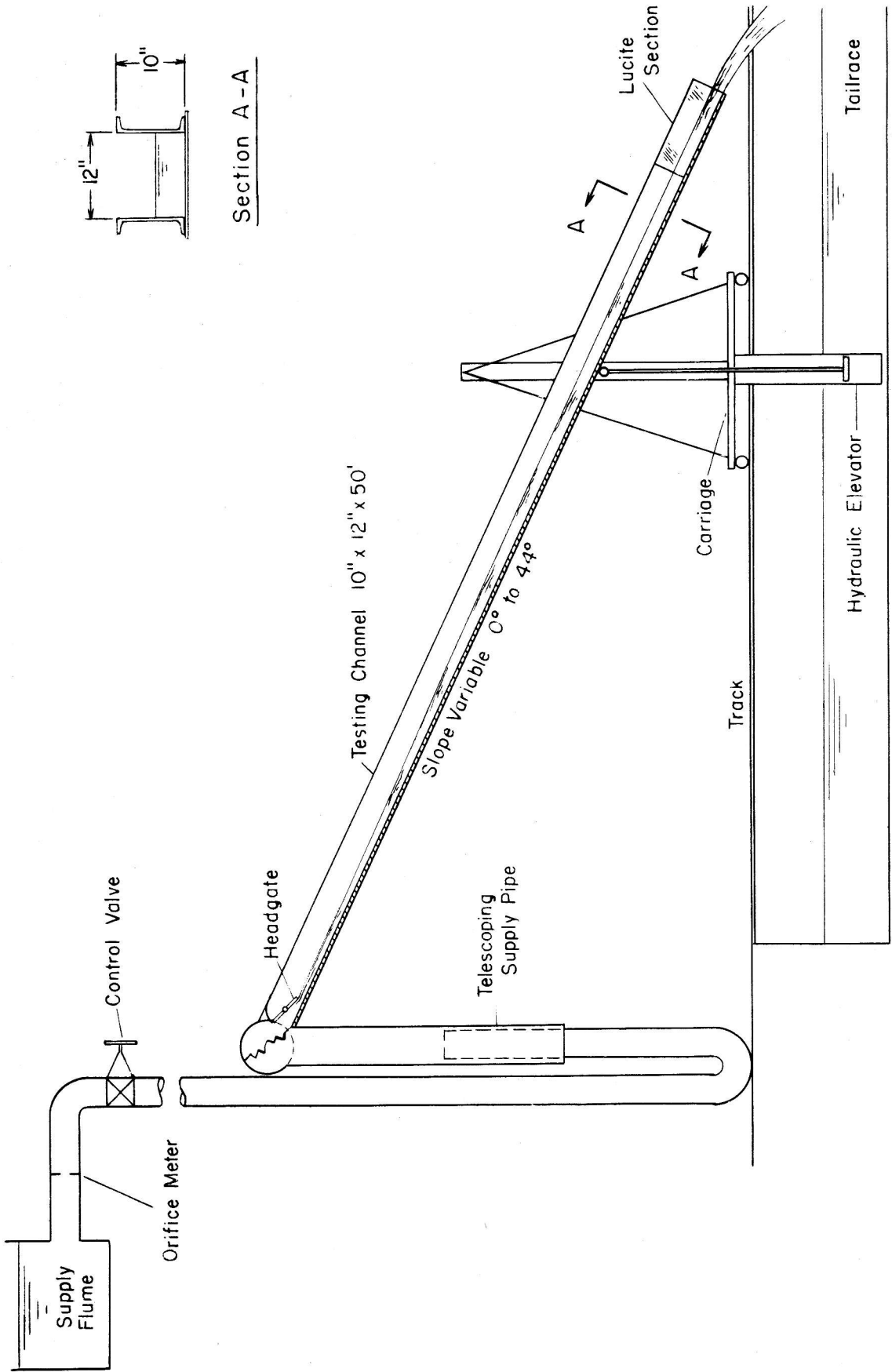


Fig. 1 - Diagram of High-Velocity Flume



Fig. 2- View of High-Velocity Flume at a 20° Slope

bulk flow direction relative to the average velocity of the underlying flow of water and entrained air bubbles. Thus the mean air concentration in a plane parallel to the channel bottom in this roughened-surface range is an average of the depressions, where there is almost 100 per cent air, and the crests, which have roughly the same air content in the form of bubbles as the underlying flow. It is probable, however, that many of these uneven disturbances persist far below the 60 per cent plane of mean air concentration and must also be accounted for in the measurement of air concentration at the lower elevations.

If air distribution and velocity distribution in a mixed-flow cross section were approximately the same, it would be possible to obtain useful information from bulk flow measurements of the air and water mixture. In most cases the distributions are known to be entirely different, making it necessary to measure air concentration as well as velocity distribution at several points in each cross section. With different distributions it is apparent that the percentage of air to water flowing through a cross section will differ from the percentage of air to water in a cross section even though the flow due to air-bubble buoyancy be neglected.

This paper deals with the design of an instrument to measure air concentration. The measurement of velocity in an air-entrained stream is being considered in a separate study.

With few exceptions, former experimental programs investigated the bulk air content and made no provision for defining the distribution of air in the flow. Total air content in the flow was defined as the deficiency between the product of mean velocity with observed cross-sectional area and an accurately measured quantity of water flow. Measured values of cross-sectional area and mean velocity were subject to large errors both in definition and in measuring technique. Any deviation in the actual flow from uniform distribution of velocity and air concentration introduced erroneous values of total air content.

A pioneering effort to define and obtain the distribution of air in self-aerated flows was made by Ehrenberger in laboratory and field experiments in Austria [2]. After an initial measurement and estimate of the vertical velocity distribution, he employed a Pitot tube to obtain a vertical distribution of stagnation heads. Assuming air to have no density relative to water, the deficiency between the measured stagnation head and the stagnation head

of water flowing at the expected velocity became the index of air concentration at the point of measurement. The velocity distribution he utilized was dependent upon the assumption of a straight-line variation between a measured velocity obtained near the channel floor with a Pitot tube (no appreciable quantity of air present at this level) and a measured velocity near the surface obtained with surface floats. The dependency of this method upon an improbable assumption of linear velocity distribution and upon a crude, surface float technique prevents satisfactory evaluation of the experimental errors.

During previous investigations of self-aerated flows at the St. Anthony Falls Hydraulic Laboratory, an instrument designed particularly to obtain traverses of air concentration was developed and used extensively. This instrument will subsequently be referred to as the mechanical sampler. Figure 3 is a photograph of the apparatus and Fig. 4 contains a diagrammatic sketch of the flow circuit. Because of the important role the mechanical sampler played in the development and verification of the proposed electrical method, a description of the apparatus and method of operation is given in the Appendix.

The principal disadvantages of the mechanical sampler are that (1) the physical dimensions of the device limit the range in which accurate readings may be obtained; that is, the air chamber fills in a very short interval of time at high air concentrations and does not fill uniformly when the air concentration is low, (2) the time necessary for a trained observer to obtain the correct aspiration setting and to balance the system averaged about 30 minutes for every point, and (3) there is no opportunity to study short time variations in air concentration with this system. Despite these shortcomings, the device gives a straightforward measurement of the mean air concentration at a point in the flow that must serve in the absence of an absolute measurement.

With this background, it is possible to state the desirable characteristics of an air-concentration measuring device. With regard to the measurement itself, an adequate definition of content and distribution of air flowing in air-water mixtures would include both the distribution of air in a cross section and temporal variations of air content at the points of measurement. The portion of the cross-sectional area included in a single measurement (the so-called point of measurement) must be small in relation to the entire cross-sectional area if an accurately defined distribution is desired. On the other

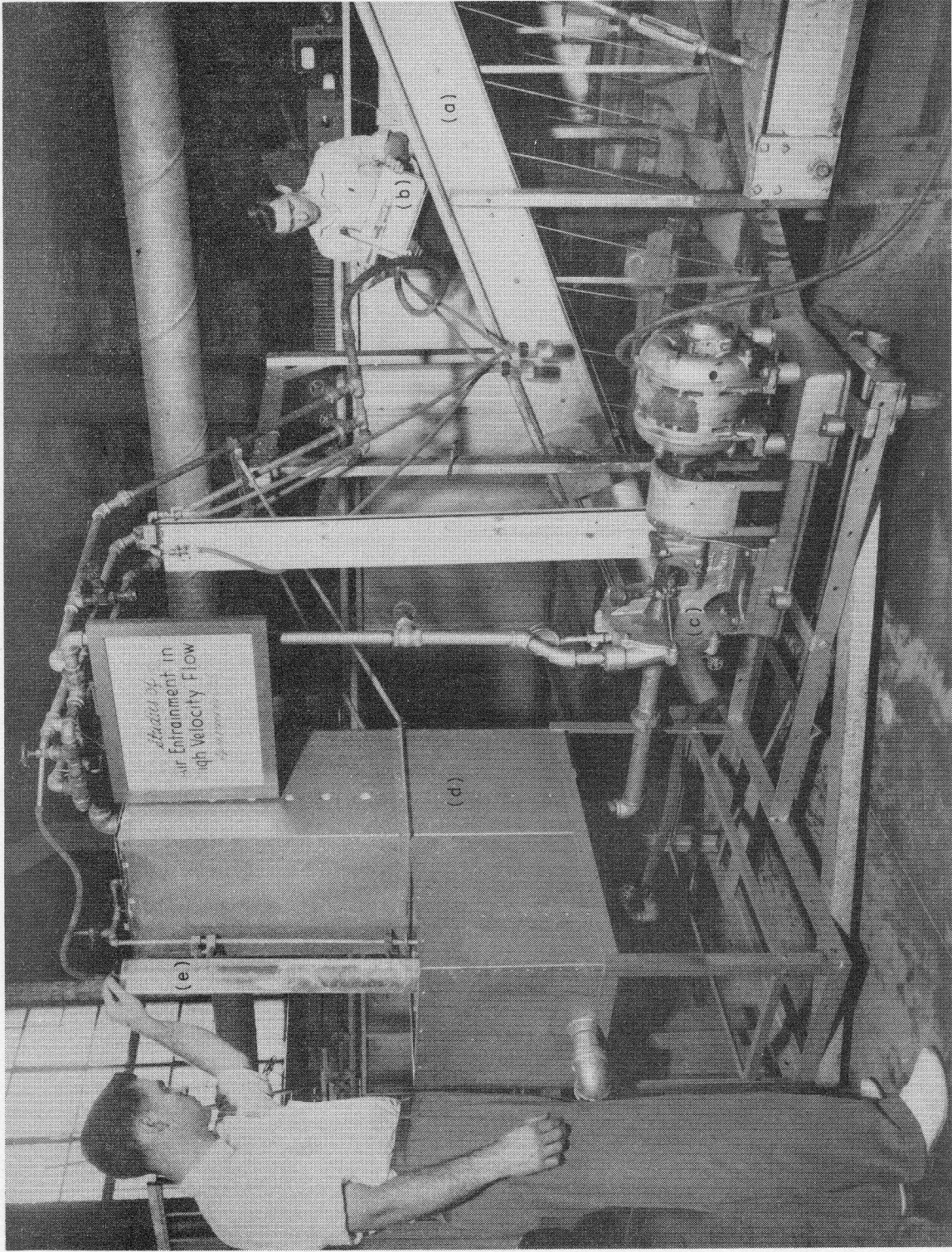


Fig. 3 - General View of Mechanical Sampling Apparatus

(a) High-velocity flume (b) Sampling tube (c) Centrifugal pump (d) Air-water separation tank (e) Air-measuring cylinder



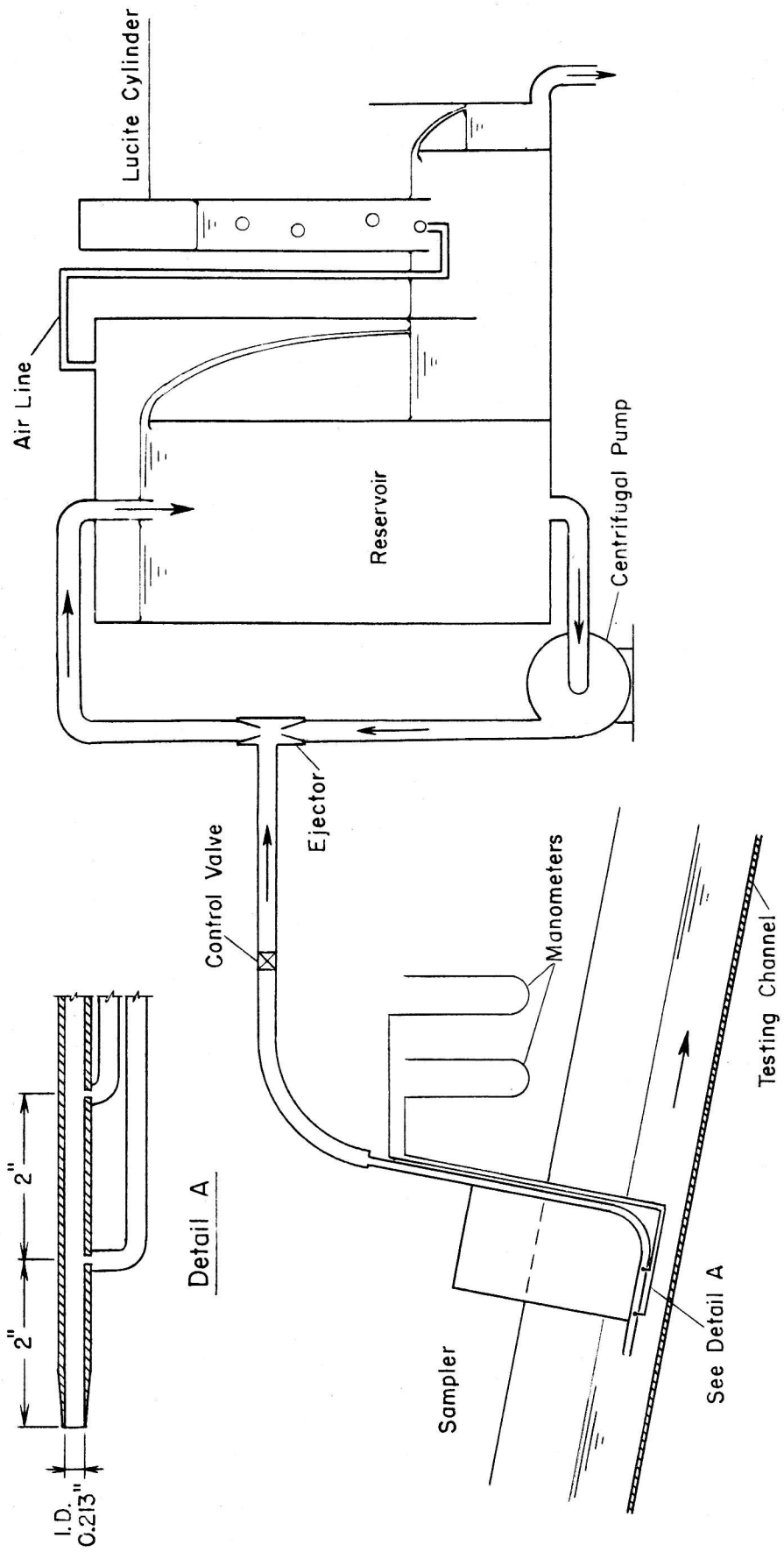


Fig. 4 – Schematic Sketch of Mechanical Sampler for Air-Concentration Measurement

hand, the area of individual measurement must be large enough to accommodate several air bubbles if a condition of full-scale variation of the measuring instrument at every bubble passage is to be avoided.

Even though the point of measurement is large enough to include several bubble diameters, there will be an appreciable change in the percentage of air concentration with the passage of each bubble past this point. These rapid changes in concentration must be averaged by the measuring instrument so that the average concentration at the point of measurement is obtained. The instrument sensitivity depends upon its ability to detect small shifts in the mean concentration at a point. An instrument that is relatively independent of calibration and is not overly sensitive to other variables is particularly desirable for the measurement of air content since there are no absolute standards of comparison that may be used as a control on the experimental work.

The high velocity of streams where self-aeration occurs requires the use of a rugged and fairly simple element for insertion into the flow. The shape of this element must not unduly disturb the flow filament being measured, as it is evident that any enforced curvature of the flowing mixture would tend to separate the air from the water because of the difference in densities. Simplicity of operation, freedom from observer errors, and conservation of time are obvious advantages in any method of measurement. An electrical method appeared to offer the best prospect of fulfilling these desirable characteristics.

## II. APPLICATIONS OF ELECTRICAL MEASURING TECHNIQUES TO RELATED PROBLEMS

The electrical method of measuring air concentration in flowing water is an adaption of methods used extensively in biological investigations of disperse systems, such as in studies of erythrocytes and colloidal suspensions.

The method depends upon the fact that the electrical conductivity of a suspension of particles in a fluid medium will vary directly with the total volume of the suspended particles. If the specific resistivity of the suspended particles is greater than the specific resistivity of the suspending medium, the resistance of the suspension will increase as the relative volume of the suspended material increases; and if the suspended material has a

specific resistivity less than that of the suspending medium, the resistance will decrease as the relative volume of the suspended material increases. It is known [10] that the conductivity of the suspension also depends upon the shape and orientation of the suspended particles. This relationship assumes importance in investigations of erythrocytes which are ordinarily discoidal in shape. Variations of resistance with temperature and ionization are definite problems in all resistance measurements of suspensions, and means must be employed to cause these factors to remain constant or their effects to be accountable.

An expression for the conductivity of a suspension of homogeneous nonpolarizable spheres was given by Maxwell [7] with the equation

$$\frac{r - r_1}{2r + r_1} = p \frac{r_2 - r_1}{2r_2 + r_1} \quad (1)$$

where  $r_1$  is the specific resistivity of the suspending liquid,  $r_2$  is the specific resistivity of the suspended spheres,  $r$  is the specific resistivity of the suspension, and  $p$  is the volume concentration of the suspended material. The derivation of this equation assumes that the spheres are spaced at such a distance from each other that their individual effect in disturbing the course of the electrical current is independent of the disturbance caused by neighboring particles.

In the case of nonconducting spheres, the right-hand side of Eq. (1) becomes equal to  $p/2$  and the solution for volume concentration of the suspended material is

$$p = \frac{\frac{r}{r_1} - 1}{\frac{r}{r_1} + \frac{1}{2}} \quad (2)$$

If the specific resistivities are taken over the same volume, electrical resistances may be substituted in the ratios for the resistivity values.

In an experimental study with cream, Fricke and Morse [4] verified the above equation within 0.5 per cent for concentrations from 8 per cent to 62 per cent. Their method of verification was to prepare samples of cream of different butterfat concentrations by exact dilutions of heavy cream and

to compare this with concentrations obtained by electrical conductivity measurement. They measured resistance of the contents of a special cell with a Wheatstone bridge, employing a substitution method at a frequency of 83,000 cycles. Precautions were taken to minimize the effect of chemical change in the cream, polarization, and other effects. The special conductivity cell consisted of a short, horizontal, cylindrical vessel mounted on pivots which would permit it to be rocked back and forth on a central axis perpendicular to its own axis. By this motion a nonconducting bead within the cell, which nearly fills its cross section, is caused to travel back and forth to stir the sample. Platinized electrodes placed in the axis of motion were recessed slightly so as not to be touched by the bead, and the temperature of the cell was held constant by a mercury temperature bath. A rise in conductivity was noted when stirring ceased, which subsided when it was resumed. This was postulated as due to the particles taking up an ordered arrangement. A change of resistance of about 2 per cent was reported as the frequency was varied from 1600 cycles to 200,000 cycles. This change was probably due to polarization and was not expected to introduce any systematic error, since the polarization is proportional to the resistance of cream of varying concentrations.

Fricke [3] extended the theory to include the case of the oblate and prolate spheroid. The relationship derived by him in this instance is

$$\frac{\frac{r_1}{r} - 1}{\frac{r_1}{r} + h} = p \frac{\frac{r_1}{r_2} - 1}{\frac{r_1}{r_2} + h} \quad (3)$$

where  $r$ ,  $r_1$ , and  $r_2$  are again the specific resistivities of the suspension, the suspending liquid, and the suspended particles, respectively;  $p$  is the volume concentration of the suspended particles; and  $h$  is a function of the ratio  $r_2/r_1$  and the axis ratio,  $a/b$ , of the spheroid. In the case of the sphere,  $h$  equals 2; if  $r_2 \rightarrow \infty$ , the equation reduces to Eq. (2).

Fricke plotted curves for  $h$  as a function of  $r_1/r$  for a number of ratios of  $a/b$  for both the oblate and prolate spheroid. If  $r_2$  is very high,  $h$  is a function of  $a/b$  only. He also applied Eq. (3) to results of other experimenters who measured the resistance as a function of concentration for blood as well as suspensions of sands in gelatine and found the value of  $h$  to be quite constant for any one experimental condition.

In passing it might be noticed that if the suspended particles are conducting and are encased in a nonconducting membrane, as in the case of many biological cells or protected colloids, the impedance of the system will be reactive. The following expression is proposed by Fricke for the capacitance:

$$C = (\text{const}) a C_0 \left(1 - \frac{r_1}{r}\right)$$

where  $r$  and  $r_1$  are, as before, the specific resistivity of the suspension and the suspending medium, respectively;  $a$  is the major axis of the spheroid;  $C_0$  is the static capacity of 1 sq cm of the membrane; and the constant depends upon the axis ratio of the spheroid. Although the capacity is not a major consideration in the measurement of the concentration of a suspension of gas bubbles, it would appear to be of importance if the method were applied to sediment measurement where it might be possible for the suspended particles to become encased in a layer of absorbed gas.

In 1940 Velick and Gorin [10], in a study of blood cells, reported an investigation of the relationship between electrical conductivity of a suspension of ellipsoidal particles and volume of the suspended material. The resulting equation is of the form

$$p = \frac{\frac{r}{r_1} - 1}{\frac{r}{r_1} - 1 + j} \quad (4)$$

for the case of zero conductivity of the suspended particle. The notation is as before. The factor  $j$  accounts for different axis ratios and orientation with respect to the uniform electric field. They calculated values of  $j$  for various values of axis ratios and orientations. Selected values of  $j$  are presented in Table I to show the magnitude of the error that any deviation from the assumed spherical shape of the bubble would induce.

Recently Morgan and Pirson [8] have employed electrical conductivity measurements as an indication of concentration of mineral suspensions. Their measuring apparatus consisted of two parallel copper rods 6 cm long by 1/2 cm in diameter placed 1.8 cm apart in a cylindrical flow cell. They offered a qualitative theory which might be somewhat difficult to justify in light of the findings of the experimenters previously cited. Their method requires

TABLE I\*  
 FORM FACTORS (j) FOR VARIOUS RATIOS AND ORIENTATIONS  
 Calculated for Zero Conductivity of the Suspended Particles

Axial ratios	Random orientation	Long axis parallel to external field, particle free to rotate	Long axis perpendicular to external field, particle free to rotate	Long axis perpendicular, intermediate axis parallel	Long axis perpendicular, short axis parallel	Model
1:1:1	1.500	1.500	1.500	1.500	1.500	Sphere
2:2:1	1.589	1.308	1.729	1.308	2.150	Oblate spheroid
3:2:1	1.643	1.188	1.870	1.364	2.376	Ellipsoid
4:2:1	1.684	1.124	1.964	1.402	2.526	Ellipsoid
5:2:1	1.731	1.096	2.049	1.422	2.676	Ellipsoid
6:2:1	1.776	1.072	2.128	1.436	2.820	Ellipsoid
7:2:1	1.827	1.058	2.211	1.446	2.976	Ellipsoid
2:1:1	1.539	2.210	1.703	1.703	1.703	Rod
3:1:1	1.577	1.122	1.805	1.805	1.805	Rod

\*Taken from Velick and Gorin [10]

calibration against known samples before it can be used. An attempt has been made to fit these experimental results to the theories of Fricke and of Velick and Gorin with little success, although the theory should also apply in this case.

### III. DEVELOPMENT OF THE ELECTRICAL METHOD OF MEASURING AIR CONCENTRATION

Consideration of air-entrained flow as a suspension of spherical, nonconducting particles in a conducting liquid was deemed valid from the observations stated in the preceding sections. The range of air-bubble concentration is probably greater than the concentration in any of the previous tests cited, but similar methods should be applicable over at least a portion of the concentration range.

The method of measuring electrical conductivity used extensively in physical chemistry studies employs a Wheatstone bridge excited by a high-frequency oscillator. A container or cell having electrodes in the wall encloses the liquid or suspension being measured and leads are furnished to connect the electrodes to the bridge. The cells are designed to minimize capacity by spacing the electrodes far apart and to minimize polarization by selecting a suitable material and a large area for the electrodes. Finally, a high degree of accuracy depends upon the ability to maintain the system at a constant temperature. Bridges and cells of these specific types are described in reports by Jones and Bollinger [5], Shedlovsky [9], and others.

With an awareness of the magnitudes of the factors found in air-entrained, high-velocity streams, it is obvious that the proved methods of the physical chemistry research studies are not adequate in themselves and can serve only as a framework for additional development. There is no precise control of temperature and homogeneity of the mixture in the flowing, entrained streams, but these considerations become relatively unimportant in the practical case where the ratio of two resistances is sufficient for the definition of concentration. However, the time between initial and final resistance measurements must be limited to a period which would not include a mixture or temperature change large enough to affect materially the ratio of resistance. This condition was found to be readily attainable during the course of the development.

Since shunt capacity increases with increasing area of the electrodes and polarization effects increase as the electrode area is reduced, it was necessary to compromise on the electrode size, aside from any other considerations such as structural strength and desired subdivision of the channel in depth. Shunt capacity entered again as the distance between electrodes was decreased to subdivide the channel in width. By using frequencies of about 3500 to 4000 cycles, the polarization effects were made small and the frequency was still low enough to prevent appreciable shunting by the capacitive reactance of the system. A further reduction of the effects of polarization could be accomplished by using platinized electrodes, but stainless steel proved satisfactory for the present purpose.

Additional considerations in the determination of electrode size and spacing are the desire for both a uniform field between plates and a large enough included volume to give a mean value of concentration; that is, the conductivity across the gap should not shift over the full scale during the passage of a single air bubble. The uniform field can be best achieved by a large electrode area and close spacing, while the distribution of the random mixture can be best measured with a larger electrode spacing, giving a volume that includes several bubble thicknesses.

In addition to the above compromises between the electrical and physical properties, the exploring device must have suitable hydraulic characteristics. For electrodes, it was proposed to attach two small, parallel, conducting plates to the end of a strut, projecting forward from the strut into the flow. The plates must be small enough to give a measurement over a very limited region of flow and cause a minimum of disturbance and yet possess sufficient rigidity to prevent excessive vibration. (Vibration caused an unexplained error during the early stages of the development.) The plates must be insulated from everything except the water on the inner surfaces and must have waterproof leads to them from the electrical instruments. Although a baked-on enamel might be superior, the use of General Electric Glyptal was both expeditious and satisfactory for insulation of the plate surface.

The strut supporting the electrodes was originally fashioned from nonconductive materials such as well-shellacked wood and Lucite (Figs. 6a and 6b). Although this measure was taken to insure against any leaks between the electrodes and their respective electrical leads, it became apparent that the large frontal area of these strut styles caused an undue disturbance in the



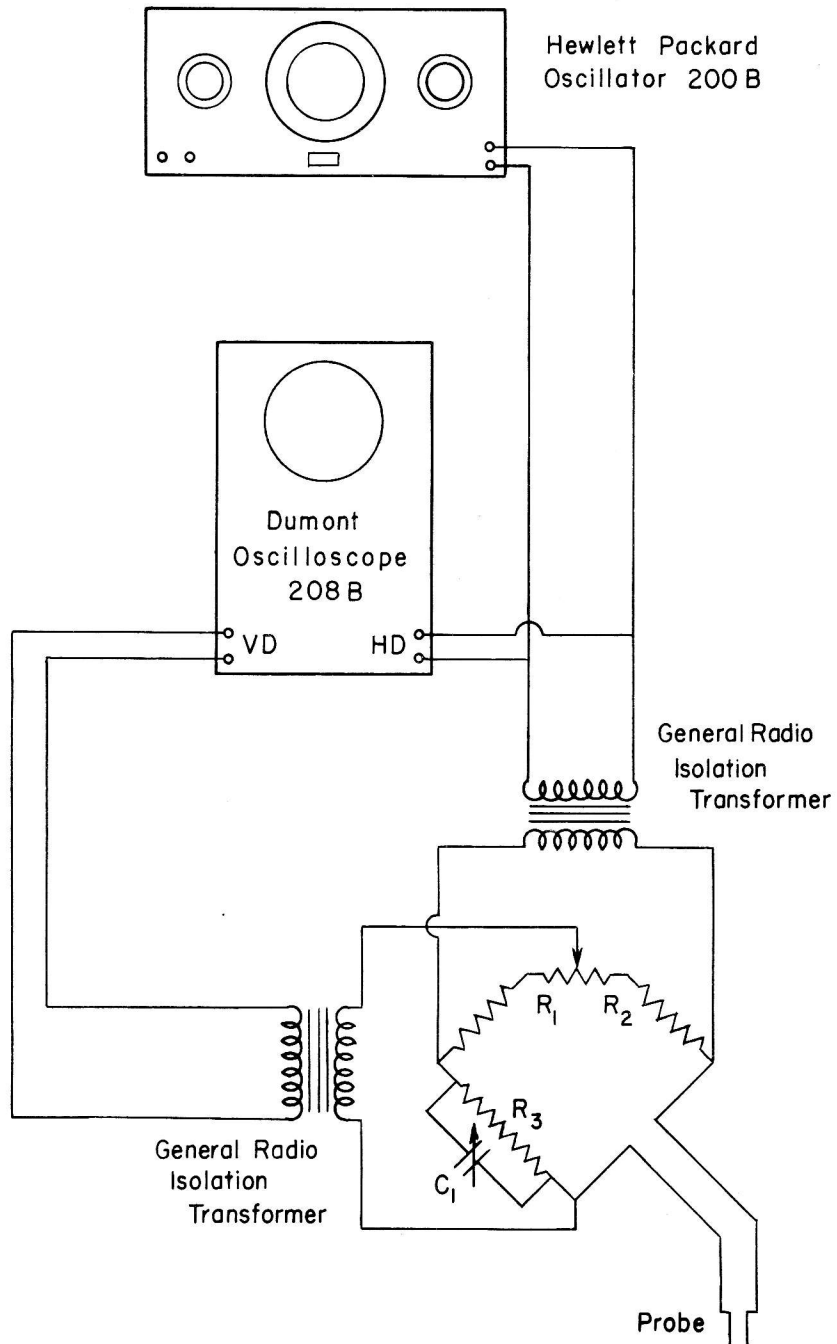


Fig. 5. — Electrical Circuit Used to Obtain Record of Instantaneous Values of Air Concentration in a Filament of Flowing Air-Water Mixture

fluid flow. The final design (Fig. 6c) employed a steel plate to give the desired structural strength and minimize the area in the path of the flow. The electrodes and leads were set into Lucite rods attached to the plate (Fig. 7) in a manner that would assure maintenance of a high insulation between them. The diameter of the Lucite rod attached to the bottom of the strut was made greater than the thickness of the plate to accommodate the full spacing of the electrodes. This bulb effect proved beneficial generally to the overall flow pattern around the strut by suppressing the surface wave that formed at the leading edge.

The usual method of measuring resistance with a Wheatstone bridge was not satisfactory in the air-entrained streams. The wide and very rapid fluctuations of air content made a bridge too difficult to balance at an average value. A voltmeter-ammeter method of resistance measurement proved more workable and had sufficient accuracy for this case. With this method the convenience of using a single instrument to measure both the mean concentration and the temporal variations in air content could be realized without any loss in accuracy.

With the single instrument method, a high-impedance voltmeter is placed across the probes and a precision variable resistor (a General Radio 0-1000-ohm resistance box in 1-ohm steps) is placed in series with the voltage source. When this resistor is adjusted so that the voltmeter reads one-half of the supply voltage, the series resistor will have a value equal to that of the equivalent impedance of the probes; this impedance is resistant for all practical purposes. To simplify the problem, a voltmeter that will take a simple average of the voltage fluctuation should be used. The type of calibration is unimportant since ratios are used. A Ballentine 300 was used in this Laboratory. If a low-impedance meter is used, shunting must be compensated. This circuit diagram is shown in Fig. 9.

A Wheatstone bridge circuit, in conjunction with an oscilloscope with proper isolation transformers, is useful for obtaining approximate minimum and maximum values of concentration. A single oscillator is used to supply both the horizontal sweep voltage for the oscilloscope and the energy to the Wheatstone bridge. The output of the bridge is connected through an isolation transformer to the vertical deflection amplifier on the oscilloscope. Under these conditions an ellipse will be visible on the screen which will degenerate into a horizontal straight line at capacitive and resistive balance of the

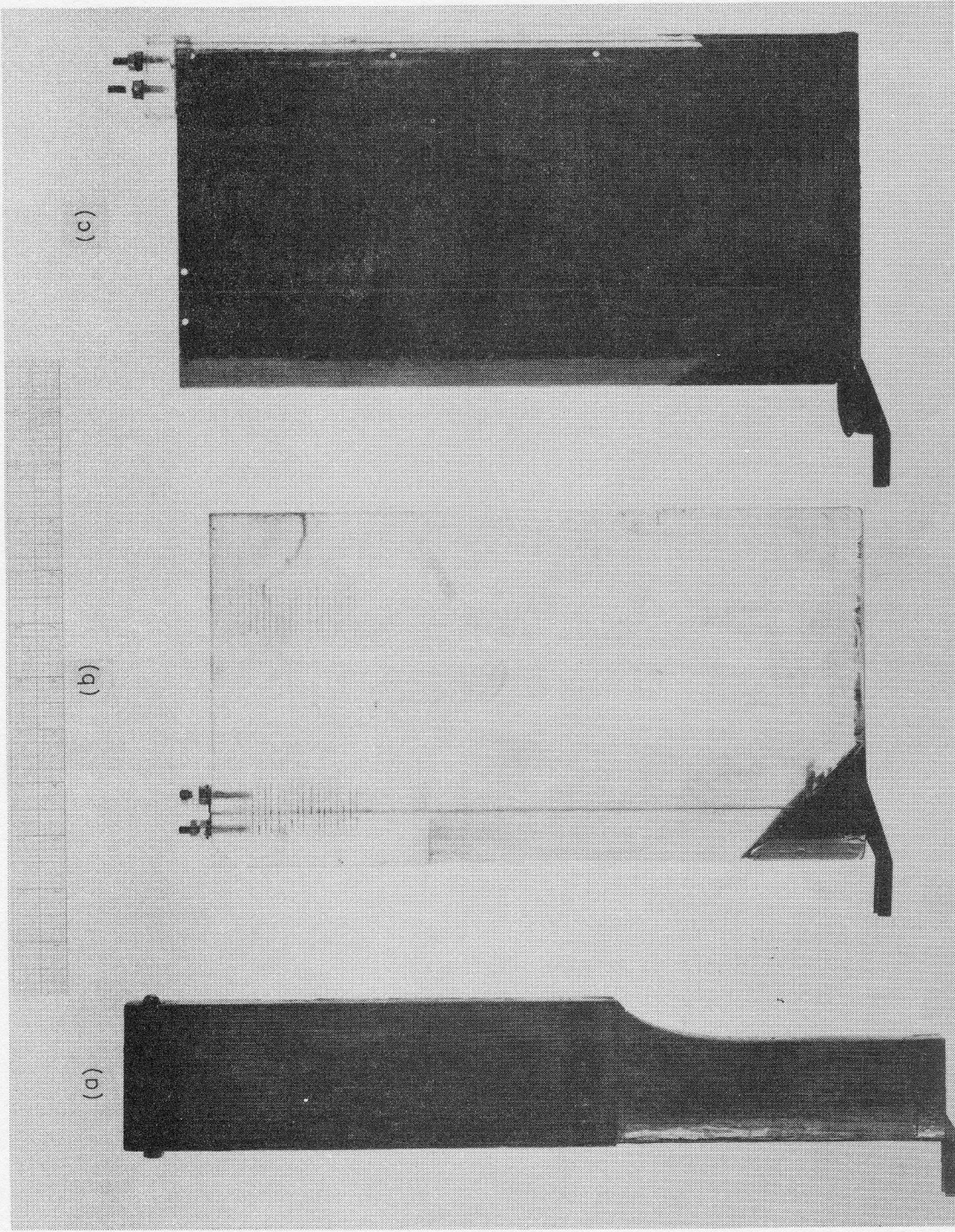


Fig. 6 - Three Probes Tested During the Studies  
Design (c) was the one selected for use in the main studies.

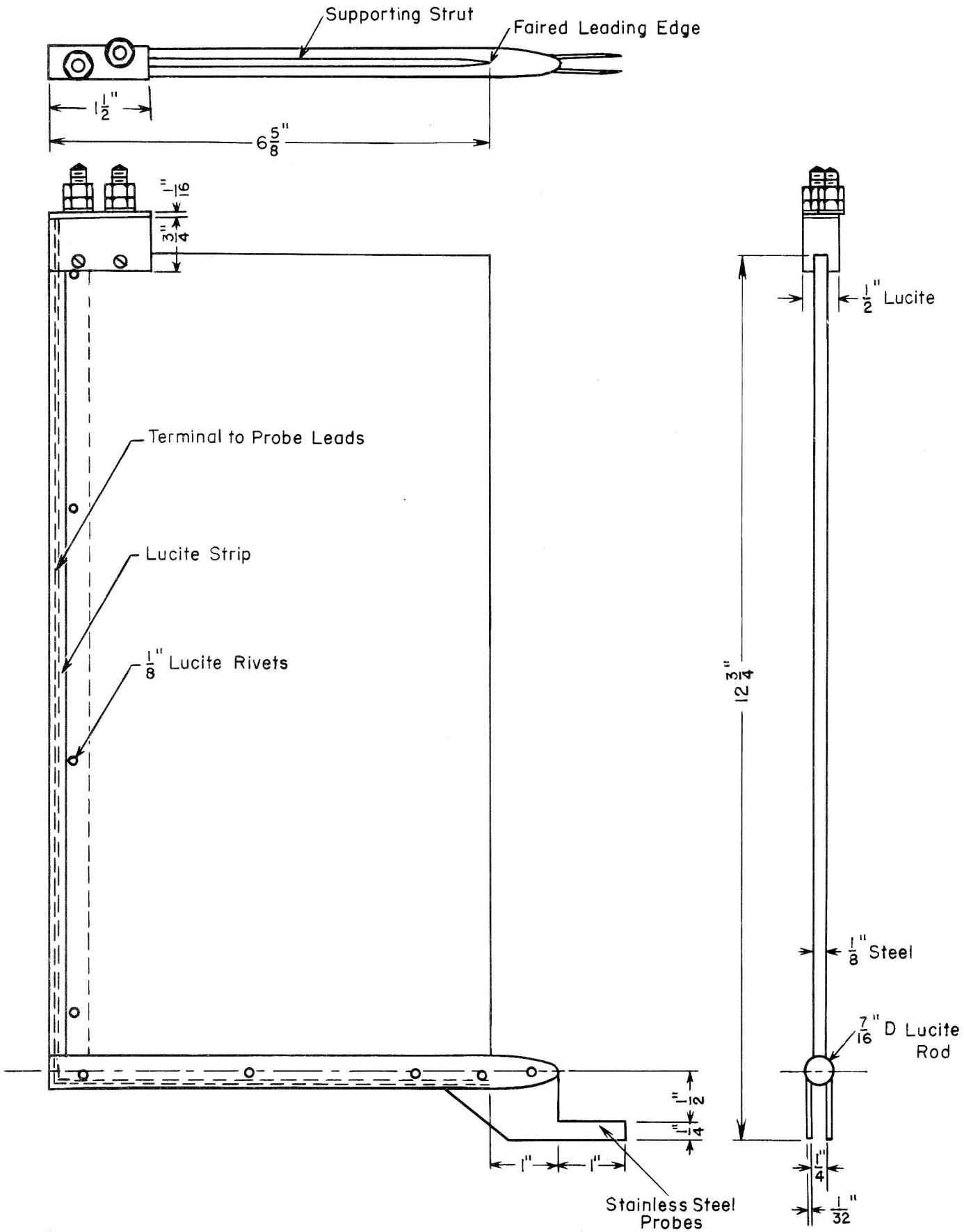
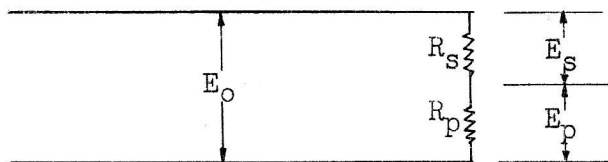


Fig.7 - Developed Form of Probes and Supporting Strut

bridge. When this system is applied to the case under discussion, the ellipse will pass rapidly through the many shapes corresponding to all resistance values. Minimum and maximum values of resistance and, consequently, concentration can be noted. This circuit diagram is shown in Fig. 5.

An investigation of the equations of resistance as a function of concentration and the current, voltage, and resistance relationship in the above voltmeter-ammeter method will illustrate a simple means of obtaining a direct-reading instrument.

In the following equivalent circuit:



$E_o$  is the supply voltage and  $R_p$  is the equivalent resistance of the probes,  $R_s$  is any arbitrary series resistance,  $E_s$  equals the voltage across  $R_s$ ,  $E_p$  is the voltage across the probes,  $I$  equals the current in the circuit due to  $E_o$ , and  $p$  is the fraction of air to total volume.

$$E_p = IR_p = E_o - IR_s$$

$$E_s = IR_s$$

$$p = \frac{\frac{R}{R_1} - 1}{\frac{R}{R_1} + \frac{1}{2}} \quad (5)$$

where  $R_1$  is again the initial resistance between the probes and suspending liquid with no air present, and  $R$  is the resistance of the mixture between the probes. Substituting

$$p = \frac{\frac{R}{R_1} - 1}{\frac{R}{R_1} + \frac{1}{2}} = \frac{\frac{E_p}{I} - R_1}{\frac{E_p}{I} + \frac{R_1}{2}}$$

$$p = \frac{E_p - IR_1}{E_p + \frac{IR_1}{2}} = \frac{E_o - IR_s - IR_1}{E_o - IR_s + \frac{IR_1}{2}}$$

$$p = \frac{E_o - I(R_s + R_1)}{E_o - I(R_s - \frac{R_1}{2})}$$

If  $R_s$  is adjusted so that  $R_s = R_1/2$ ,

$$p = \frac{E_o - 3IR_s}{E_o} = 1 - \frac{3E_s}{E_o} \quad (6)$$

From Eq. (6) it is seen that if the ratio of  $E_o$  to  $E_s$  is 3 at initial conditions, the condition  $R_s = R_1/2$  will be met. Also, by Eq. (6),  $E_s$  varies linearly with  $p$ . Now, if the voltage meter is made to read full scale on a 0-100 scale when the probes are immersed in water without air, regardless of the actual voltage measured, the reading will be directly in percentage of water. Furthermore, the average value of a rapidly fluctuating  $p$  will be given by the average value of  $E_s$ .

In the Laboratory a Hewlett Packard 200 B oscillator has been used to give a frequency of 4000 cycles and an output of 7 1/2 volts. The output terminals were shunted with a 50-ohm resistor to give a more constant output with varying load. A Weston 779 voltmeter was used as the indicating instrument. The high damping of this instrument made reading easy. A diagram of the circuit is shown in Fig. 9b.

In general, an isolation transformer must be used if the oscillator does not have an output transformer, if one of the output terminals is grounded or if one of the output transformers has excessive interwinding capacity. It is important that no part of the system is grounded except through the flowing water and that the measuring system is not exposed to high magnetic or electric fields.

To zero the instrument, a sample of water from the flow is collected in a pail and the probes are immersed in it while constant stirring is maintained. The meter is then set on the 2 1/2-volt scale (or any low scale) and placed

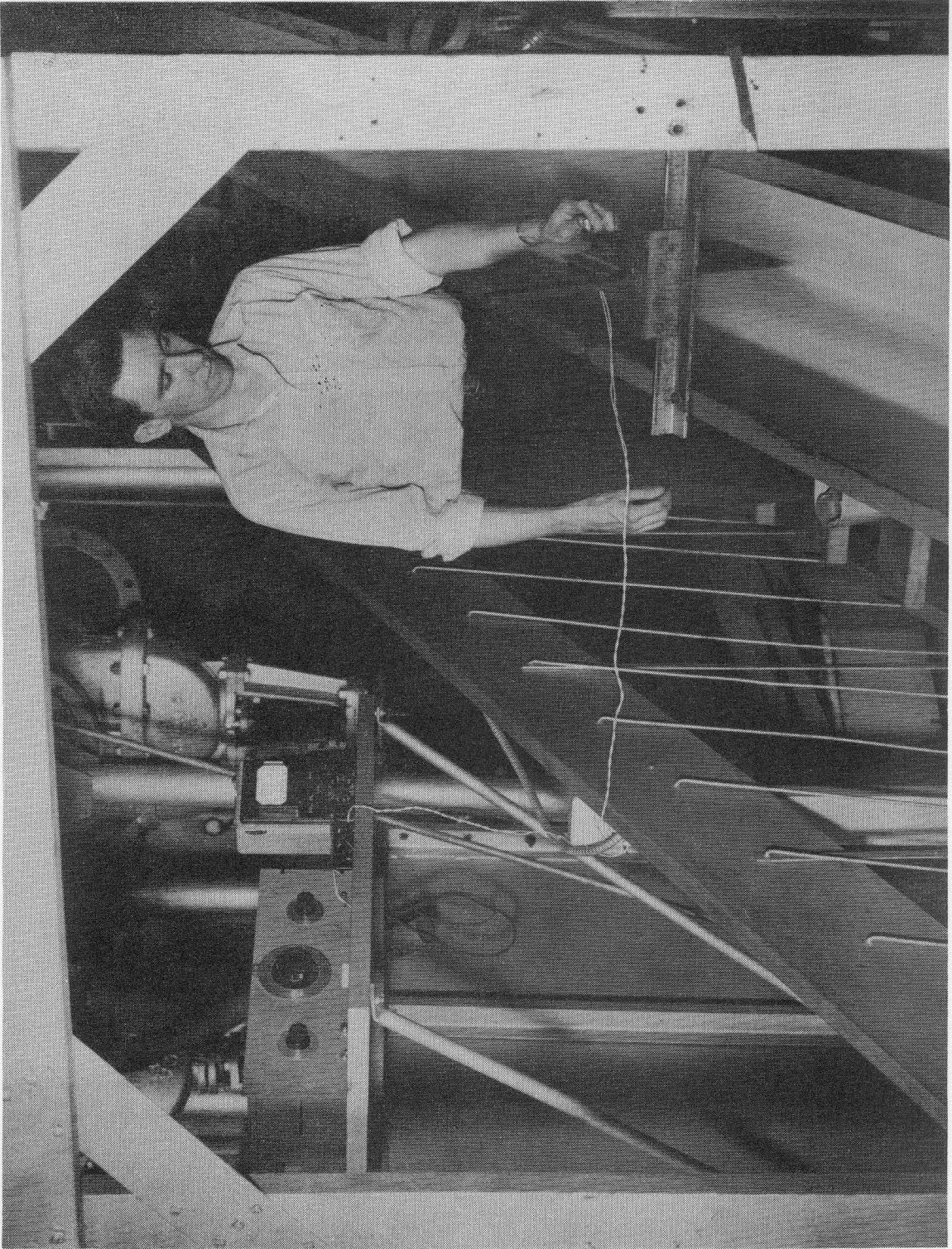


Fig. 8 - View of Electrical Probe Immersed in Flow  
The electrical apparatus used with this unit is shown in the upper left section of the photograph.

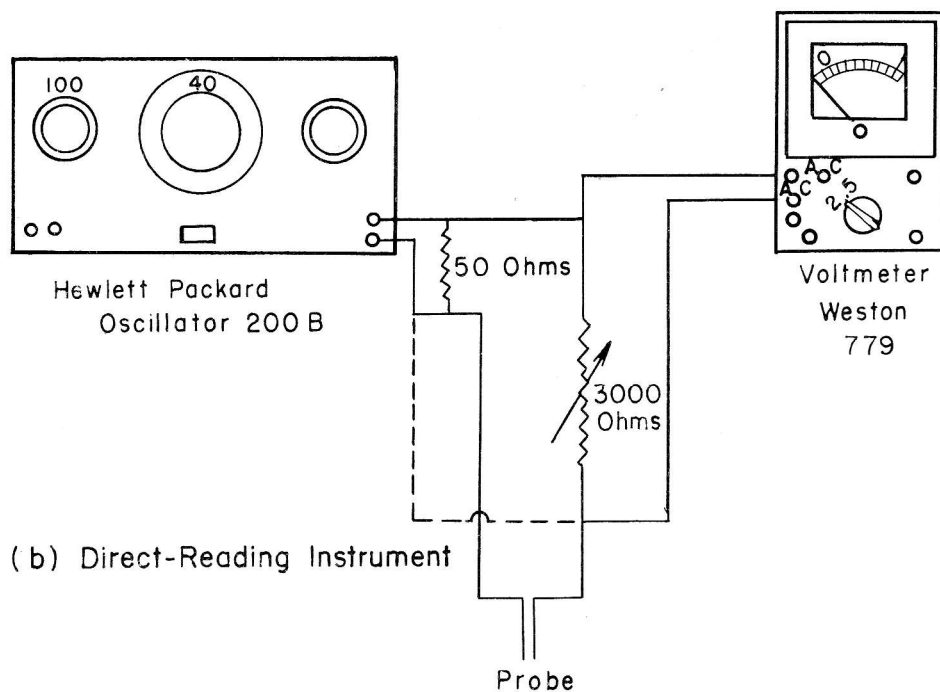
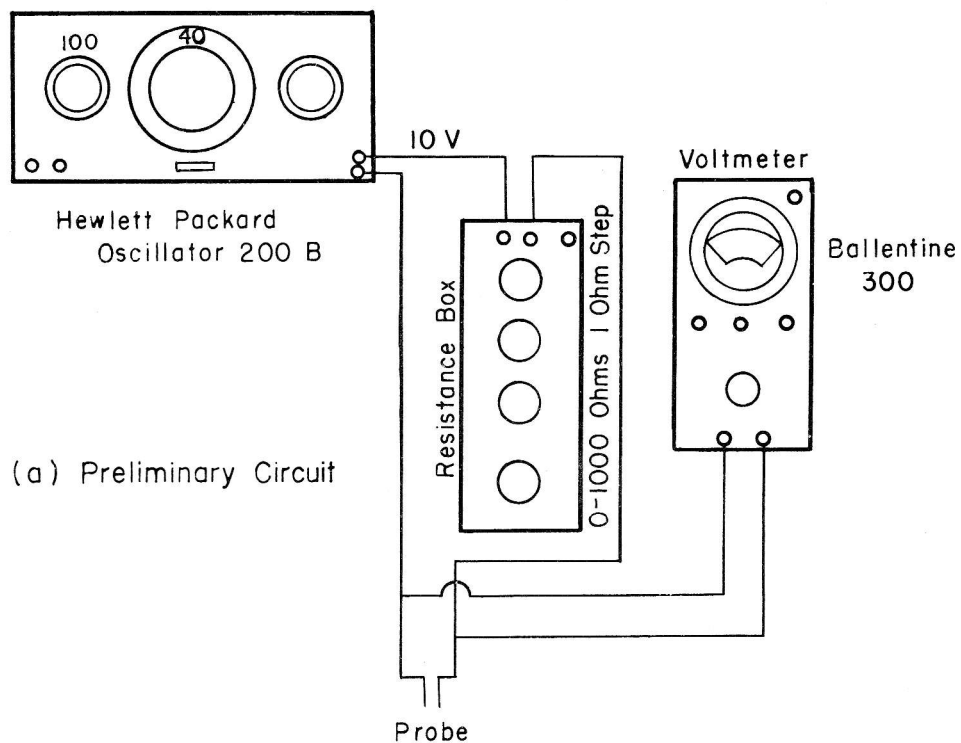


Fig. 9 - Electrical Circuits Used in the Measurement of Mean Air Content in a Filament of Flowing Air-Water Mixture



across the oscillator terminal. The output is adjusted to an arbitrary value readily divisible by three. The meter is connected across  $R_s$ , which is an ordinary carbon potentiometer, and the value of the resistance  $R_s$  is adjusted until one-third the previous value is obtained. It is important that the oscillator voltage be maintained constant. With the meter across  $R_s$ , the oscillator output is advanced until the meter reads full scale. The meter will then read percentage of water times 100 on the 1-volt scale when the probe is placed in the flow.

When the previously described voltmeter-ammeter method or the bridge method is used, the initial resistance is determined from a sample withdrawn from the flow. Then resistance for any other concentration is determined. The percentage of air is computed from the following formula:

$$p = \frac{\ell - 1}{\ell + \frac{1}{2}} (100) \quad (7)$$

where  $\ell$  is the ratio of final to initial resistance or  $R/R_1$ .

The midpoint of the vertical dimension of the probes has been assumed to be the point of measured concentration. In regions of rapidly changing resistance with depth, this assumption must be altered due to distortion of the current flow pattern. However, when the mean concentration reading is obtained from a series of rapidly changing regions of concentration, such as are found in self-aerated streams, the use of the midpoint of the vertical dimension is again valid.

The several auxiliary electrical instruments may be replaced by a single cabinet enclosing a fixed frequency audio oscillator (4000 cycles) having an isolated output and good voltage regulation; a high-impedance, 3-volt AC voltmeter with a symmetrical input (to avoid capacity unbalance of the measuring probes to ground) and inverted scale; a ganged two-step attenuator (0, 1/3E); and a meter switch to facilitate a simple zero adjustment of the meter for initial calibration.

#### IV. MEASUREMENT IN HIGH-VELOCITY FLOWS

The variable-slope testing flume at the St. Anthony Falls Hydraulic Laboratory (Figs. 1 and 2) was expressly designed for the study of self-aeration and air-entrained flows. The channel section was 50 ft long, 1 ft

wide, and 10 in. deep. The quantity of water flow and the static head at the flume inlet were controlled by a gate valve in the supply piping and a butterfly-type valve at the inlet to the 50-ft reach of open channel. With this arrangement, it was possible to jet the incoming water stream at the approximate terminal velocity for the set slope and discharge. Thus, there was little velocity change over the channel reach and the major change in the flow was due to air entrainment. For certain flow conditions it was possible to obtain a relatively stable condition of air entrainment over the last 10 to 15 ft of the open channel. Such a stable condition was used during the initial testing of the electric instrument and during the checking of the instrument against the mechanical sampling device. The test point was established at a station located 43 ft from the inlet gate.

An exhaustive review of the measuring techniques that were tried and discarded during the development of the instrument is not necessary, but the following summary of conditions that seriously influenced the concentration readings is presented. Early measurements were often influenced by a channel cross current which caused a region of separation near the exposed or inner surface of one of the electrodes. In addition to more carefully aligning the probes with the direction of flow, it was found advantageous to bend the probes slightly outward so that any separation would occur on the outer or insulated face. The resulting small contraction of the measured filament had no noticeable effect on the readings. When the probes were placed close to the surface of the flowing mixture, an obviously erroneous reading was induced as the probes broke the surface, so that the upper half of the probe was nonconducting while the lower portion retained a current flow characteristic of the particular mixture. The remedy for this condition would be a reduction in the dimension of the electrode normal to the channel bottom, but such reduction would be undesirable because of increased polarization and electric field distribution. This effect limited the proximity of measurement to the upper surface of the flow. Also, it was found that measurements closer than 0.02 ft from the channel walls were influenced by current deflections by the walls.

As the measurement of the percentage of air (Eq. 5) always depends upon the resistance or specific resistivity of the suspending fluid, it is imperative that this factor be measured accurately. Many of the early difficulties with the method were traced directly to inaccurate readings of initial liquid resistance. Attempts to obtain the resistance reading of the liquid

by lowering the probe close to the bottom of the flume were not successful because many flows, especially those at higher velocities and shallow depths, contained a high percentage of air at all measurable levels. When a sample of water is withdrawn from the flow, the entrained air will settle out in a matter of seconds. If the measurement of liquid resistance is not taken immediately after the air bubbles settle out, appreciable errors (mainly a result of settling and realignment of microscopic, solid particles, small temperature changes, and deposition of gaseous particles on the electrode faces) are certain to affect the readings.

The errors caused by electrode vibration have not yet been satisfactorily explained, but it is possible that small gaseous pockets were intermittently formed at the inner faces of the electrodes, thus causing the observed overestimate of air content at that point. Minor changes in the shape and thickness of the electrodes served to eliminate the early errors due to vibration of the electrodes.

The placement of the electrodes below and in front of the supporting strut permitted the measurement of concentration in the flow filament without interference of the supporting mechanism. However, the proximity of the electrical instruments and the consideration of observer efficiency caused a need for an electrode supporting structure that would not induce excessive splashing and spray. The strut pictured in Fig. 6c has desirable electrical properties (previous section) and satisfactorily suppressed the splashing and spray formation at most of the set positions in the flow. When the strut was set so that the Lucite bulb was at the surface of the high-velocity stream, a considerable amount of water was deflected outwards in the downstream direction. This single deficiency was not considered serious enough to warrant further investigation of supporting strut shapes.

The electrical concentration measuring instrument was checked by comparison with readings obtained on the mechanical sampler, as stated previously. Nearly all of the data from the mechanical sampler presented by DeLapp [1] from studies with the air-entrainment channel were centerline traverses of concentration at the station 43 ft from the inlet gate. By reference to DeLapp's traverses (which were checked by several repetitions), it was possible to analyze the reproducibility of measurements with the mechanical sampler. It was then possible to establish several standard profiles of concentration that could be used in initial checks of the electrical instrument.

When the electrical instrument was developed further, it was necessary to make actual day-to-day comparisons of the two methods. This, of course, was more time consuming, but eliminated errors due to small differences in setting the slope and discharge of the channel.

The comparison of the electrical method with the mechanical sampling method was conducted with several combinations of slope and discharge of the entrained stream. These combinations agreed with selected initial studies of the mechanism of entrainment then in progress on the steep-gradient channel.

In Fig. 10 measured values of the concentration of air are plotted against the depth of measurement for a water discharge of 4 cfs and a channel slope of  $16^\circ$ . The electrical measurements were obtained both before and after the traverse with the mechanical sampler, and no change was made in the flow during the elapsed time. An operator of the electrical instrument set the probe in the flow and obtained one traverse. An alternate observer operated the mechanical instrument. The operators of the instruments were changed during the course of the various comparison tests. Figures 11 and 12 are similar graphs showing the comparisons at  $24^\circ$  and 4 cfs and  $44^\circ$  and 3 cfs, respectively. These particular graphs are representative of a considerable number of comparison traverses made after the final form of the electrical instrument was developed.

Repeated traverses of the same flow by the electrical method, such as the traverses taken before and after the mechanical sampling traverse in the above-mentioned comparison runs, exhibited a better reproducibility of measured results than could be obtained with the direct mechanical method. The mechanical method also proved to be more dependent upon operation by a skilled observer than the electrical method when the results of various operators were analyzed.

Effective points of measurement in the plotted graphs were taken as the midpoint of the electrodes and the center of the opening in the mechanical sampling tube. The depth measurements were carefully zeroed in with the bottom of the channel each time the instruments were placed in position.

## V. ANALYSIS OF MEASUREMENTS

The theoretical development of Eq. (5), on which electrical measurements depend, was based on the assumptions of zero conductivity of the suspended air bubbles, spherical-shaped bubbles, and sufficient spacing between

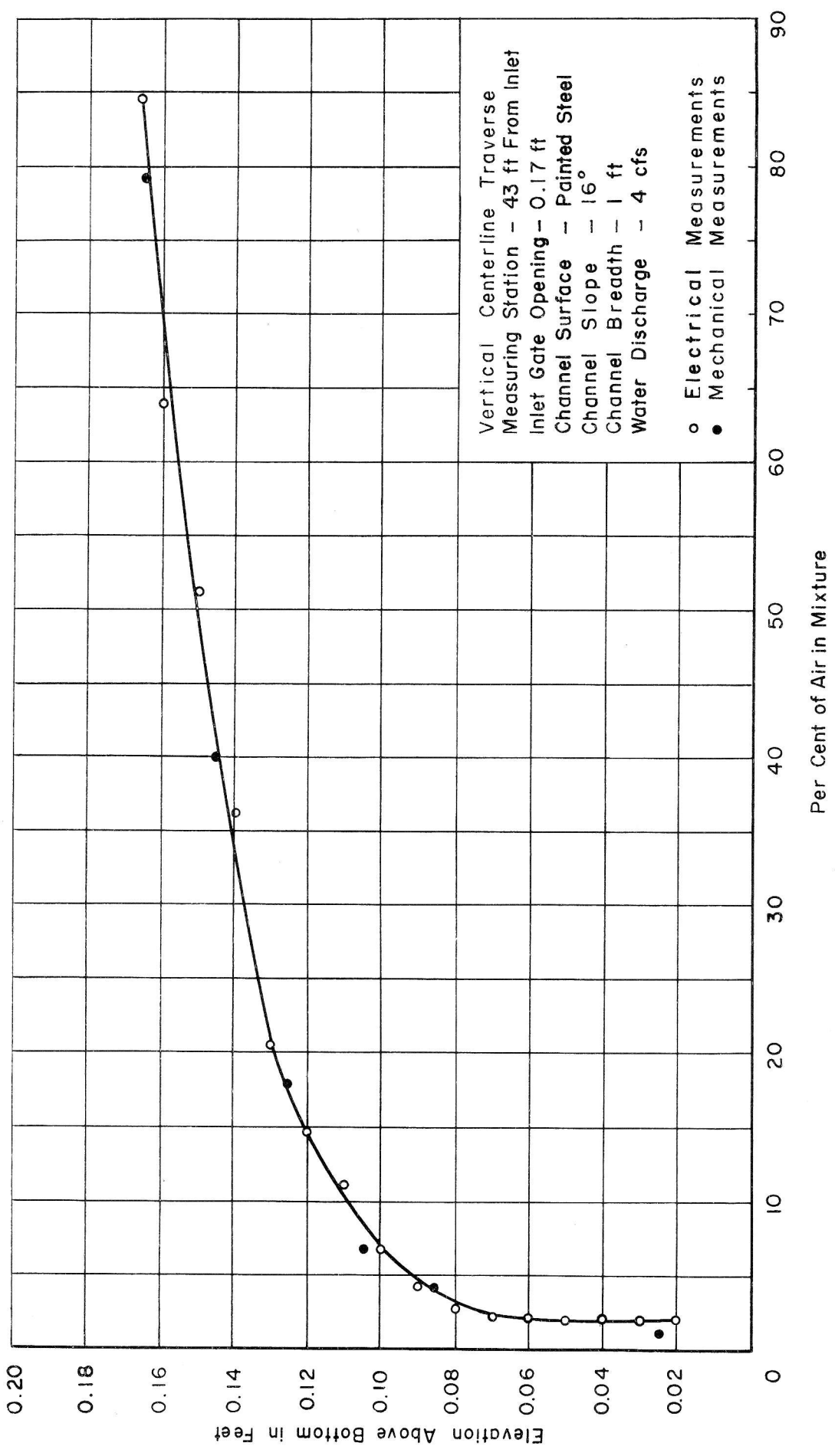


Fig. 10.— Comparison of Air-Concentration Measuring Instruments in Uniform Flow - Channel Slope 16°

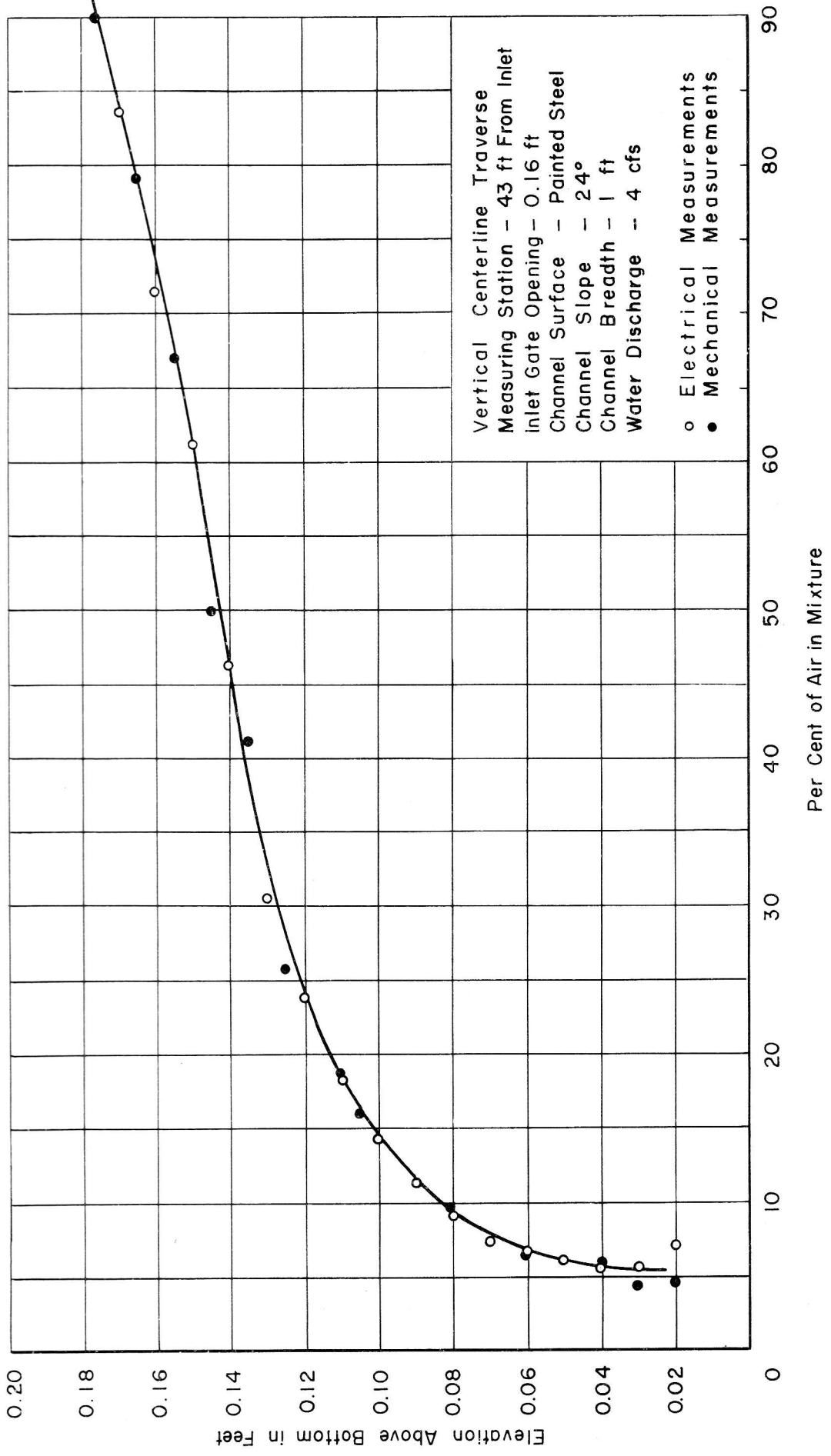


Fig. 11.- Comparison of Air-Concentration Measuring Instruments in Uniform Flow - Channel Slope 24°

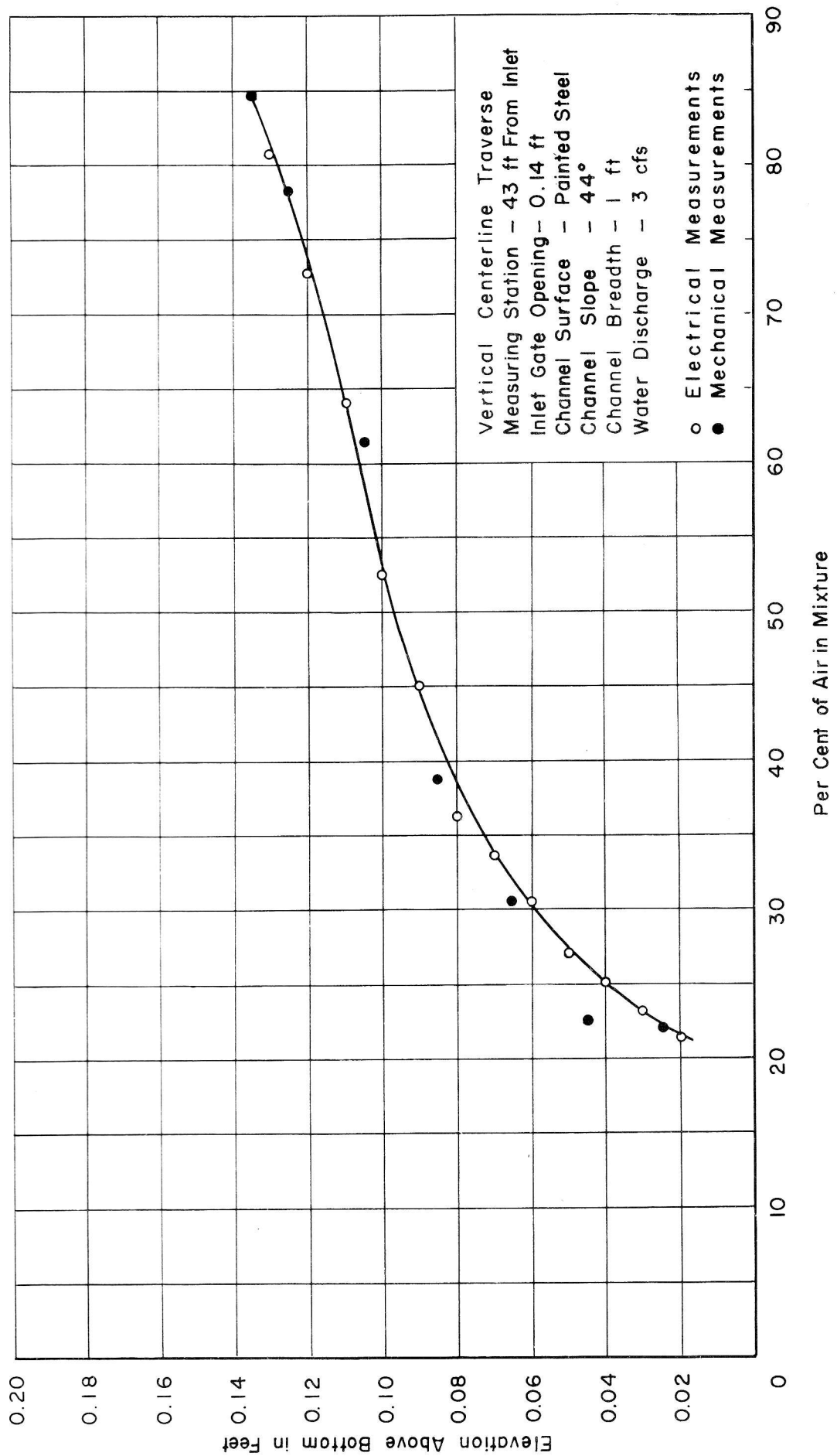


Fig. 12.- Comparison of Air-Concentration Measuring Instruments in Uniform Flow - Channel Slope 44°

bubbles to prevent interference of the current lines. The first assumption implies that continuous paths of water between the electrodes are necessary if current is to flow. A mixture of air and water in the mist stage (that is, a mixture where air encloses droplets of water) could not conduct current or have a potential across the electrodes different from air alone. The probable upper limit of air content for the foam stage, the stage where water encloses the air masses, is between 60 and 70 per cent, although soap or other film-forming conditioners would produce a foam mixture having a higher upper limit. The limit of 60 or 70 per cent was the expected upper limit of the applicability of the developed electrical method.

The assumption of freedom from mutual interference by neighboring bubbles at first seemed to be the most severe limitation on the range of air contents measurable by a direct application of the theory. However, Fricke and Morse [4], in their experimental work with prepared mixtures of butterfat in cream, obtained results within 0.5 per cent up to 62 per cent concentrations of butterfat.

Examination of Figs. 10, 11, and 12 reveals a close agreement between the electrical and mechanical measurements. No significant difference between the results was present at any of the measured levels of air content, although the squared deviation of the concentration values obtained by the mechanical sampler was slightly larger than the squared deviation of the electrically obtained values. A general statistical analysis of variance between the instruments, the depth settings, and the observers served only to show the dependence of the mechanical sampling method on a carefully trained observer.

Agreement between the electrical and mechanical samplers at levels of concentration above 60 per cent was not expected because of the supposed absence of electrical conducting paths at these higher concentration levels. Since agreement was obtained, however, an explanation was sought. By placing a single probe in the flow near the water surface and connecting an oscillator voltage source between the probe and a ground on the flume, a picture of the condition of the flow was obtained on an oscilloscope placed parallel to the probe and ground. A resistor was placed in series with the oscillator output to provide a varying voltage with changes in conductivity. Because of the rapid changes in the water-air mixture as it streamed past the probe, it was necessary to photograph the oscilloscope tube on a moving strip of 35 mm film. This test showed that the flow near the surface is composed of alternate slugs



of nonconducting air and conducting air-water mixtures. If the indicating instrument of the electrical method followed the rapid concentration changes perfectly, its mean reading over a period of time would yield the true value of the mean concentration (except for the small quantity of spray and droplets that might be present in the air pockets and would not be measured). The indicating instrument is an averaging-type voltmeter or, more accurately, a rectifying AC voltmeter. It is apparent that the needle of the meter cannot follow the rapid fluctuations of voltage (20 to 100 per sec) from full voltage to some lower value indicative of the instantaneous current flow, but that it will present some average of the varying voltages which probably differs from the true mean. The close agreement between the electrically and mechanically measured values at the upper levels of concentration, however, is considered sufficient proof that the average needle reading is a satisfactory approximation to the true mean voltage. The mechanical sampler is not subject to the same averaging problems.

The electrical method was used to measure air concentrations as high as 90 per cent with good accuracy, as may be seen in Fig. 11. Above this level, accuracy is not to be expected. The layer of flow near the surface containing 95 per cent or more of air in a stream whose total depth is less than 6 in. is so thin that the probes will extend through the surface and erroneous readings will be obtained for reasons previously stated.

### CONCLUSIONS

In conclusion, the following characteristics and advantages of the electrical air-concentration measuring method developed at the St. Anthony Falls Hydraulic Laboratory may be stated:

1. Good accuracy is obtained with the electrical method at all concentration levels to over 90 per cent, as shown by the agreement with data obtained in a mechanical sampler (Figs. 10, 11, and 12).
2. The electrical instrument can be used efficiently to obtain more complete air-content traverses of high-velocity, self-aerated streams than are now available from any other method.
3. It is not necessary to use a calibration factor with the electrical measuring equipment; the mechanical sampler data cited

in the body of the report were used only to verify the results of the electrical method.

4. The linear relationship between voltage and concentration makes it possible to use the electrical instrument in a direct-reading form independent of graphs and charts.

5. The time required to obtain an electrical measurement of the air concentration at a point in the flow averaged about 30 seconds during the later phases of the developmental work. (It averaged about 30 minutes with the mechanical method.)

6. By use of a more rugged strut and supporting structure the instrument could be applied to large flumes and spillways.

The measuring probe finally adopted is shown in Figs. 6c and 7, and the electrical circuit is shown in Fig. 9b.

#### ACKNOWLEDGMENT

The work on air-concentration measurement was performed in connection with the study of self-aerated flow sponsored by the Office of Naval Research under Contract N6onr-246. The mechanical sampling device which proved invaluable in the verification of the present method was developed by John F. Ripken, and the paper was reviewed critically by Edward Silberman.

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A P P E N D I X

## MECHANICAL SAMPLER

The following description of the mechanical sampler is taken verbatim from a thesis by DeLapp [1]. The description should be read with Figs. 3 and 4 in view.

"In order to determine the ratio of air to water in a flow, a sample of the mixture is continuously drawn off into a closed tank and after separation the rate of flow of each is determined. The sampler consists of a brass tube with an inside diameter of 0.213 in. mounted on a steel plate for rigidity, and held securely in the test channel with the open end of the tube pointing upstream. At points 2 and 4 in. from the open end of the tube a small hole is drilled through the tube perpendicular to its axis, and copper tubing attached. This tubing led to open manometers so that the pressure at each of these points in the sampler could be determined during the sampling process. The purpose of this arrangement was to indicate the proper rate of sampling so as to result in a minimum of disturbance to the flow. Since the pressure drop between the two gage connections should be the same as between the end of the tube and the first connection, it is evident that by adjusting the rate of sampling until the pressure at the first connection is less than atmospheric by the amount of difference in pressure between the two connections, the pressure at the end of the tube should be exactly atmospheric and no impact or curvature of the streamlines should result.

"The vacuum necessary for drawing off the samples was provided by three ejectors connected in parallel. Pipe lines led from each of these and were each provided with control valves and were joined at a common point before connecting with the sampling tube. The rate of sampling could thus be varied over a wide range by adjusting the control valves and employing one or more ejectors as needed. The ejectors were supplied by a centrifugal pump drawing water from the same closed tank into which the ejectors discharged, so that the only water added to the system was that drawn from the test channel by the sampler and ejector.

"After discharging into the closed tank the air and water separated, the latter discharging from the tank through a calibrated submerged orifice. A partial partition in the closed tank maintained a constant head on the pump and reduced the time necessary for the head on the orifice to become constant when the rate of flow was changed. Any one of four orifices could be used to discharge the flow from the tank. These had diameters of 1/4 in., 3/8 in., 1/2 in., and 3/4 in., allowing accurate metering of discharges ranging from 0.0013 to 0.036 cfs. The depth of water in the tank was indicated by an open piezometer.

"The air in the sample discharged from the closed tank through a 1/4-in. diameter pipe which was arranged to discharge either into

the atmosphere or into an inverted tube where the rate of air flow could be measured volumetrically over the water. This tube was transparent plastic and had a diameter of  $3 \frac{1}{8}$  in., a length of 36 in., and a volume of 0.138 cu ft. The rate of air flow was indicated by the time required to fill the tube, determined by a stop watch accurate to 0.2 second. Graduations near the end of the tube made possible the determination of the air volume to the nearest 0.001 cu ft. At the end of each run the air was exhausted from the tube and the water replenished by a small auxiliary line from the top of the tube to one of the ejectors."

A number of traverses of air concentration obtained with this sampling instrument are presented in the thesis. In the current program the water discharge from the apparatus was weighed over a set time interval and also metered by the aforementioned orifice system.