

EXPERIMENTAL STUDIES OF AIR ENTRAINMENT IN  
OPEN CHANNEL FLOW

*by* Lorenz G. Straub and Owen P. Lamb

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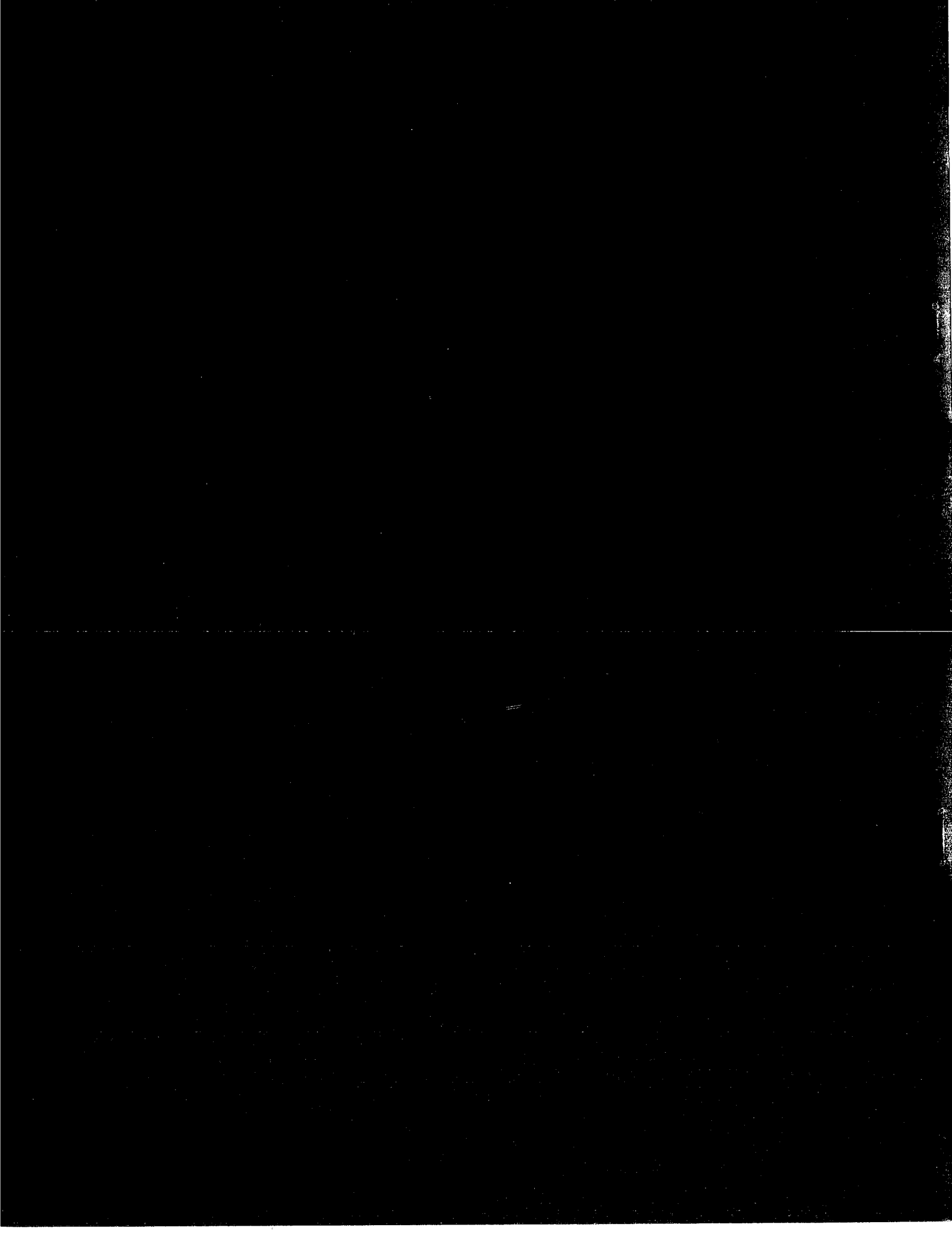
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# EXPERIMENTAL STUDIES OF AIR ENTRAINMENT IN OPEN CHANNEL FLOW

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## ABSTRACT

The phenomenon of insufflation of air into water flowing at high velocities in open channels has long been of great interest to hydraulic engineers, but detailed experimental information regarding the occurrence and the investigation of the flow mechanism of air-water mixtures has been elusive because of inadequacy of accurate observations by customary instrumentation. New instruments for measuring velocities of mixed flows and the air concentration in these flows have been devised at the St. Anthony Falls Laboratory so that velocity traverses and air entrainment traverses can now be made accurately for widely diverse flow conditions both as regards velocities and percentages of air entrained.

This paper describes the results of experimental observations of self-aerated flows in an open channel for various slopes up to 45°. It is pointed out that customary open channel flow relationships do not apply directly for air entrained flows. For the range of conditions reported, actual velocities are shown to be greater than computed velocities by hitherto proposed methods. The experiments reported are for conditions of a smooth painted steel surface. The authors contemplate further studies with channels of differing roughness and greater slopes.

## INTRODUCTION

The experimental study of self-aerated flows in flumes with steep gradients is a phase of air-water mixture studies now in progress at the St. Anthony Falls Hydraulic Laboratory. These experiments are being conducted in a large laboratory facility (Fig. 1) especially designed and constructed to provide naturally aerated flows through a considerable range of discharges and at all flume slopes from horizontal to vertical.

It was evident from the start of this research that the customary instrumentation for hydraulics and fluid flow observations would be inadequate to obtain more than quite superficial information in which much would still be wanting to avoid misinterpretation. Much attention at the Laboratory has therefore been given to devising instruments to measure the local values of air concentration and velocity in small increments of the cross-sectional areas of the flows in the experimental flume.

The method devised of obtaining air concentration measurements involves the determination of the electrical resistance of the filament of fluid mixture bounded by two small electrodes. Maxwell's equation for the resistance of a suspension of spheres in a conducting medium was utilized to initiate the design. An equivalent electric circuit was developed to adapt the expression to air entrainment conditions where the fluid mixture does not always occur as a homogeneous combination of the air and water phases. The instrument as it is now being used is accurate, direct-reading, easily handled, and is not subject to calibration by volumetric means for its operation. Design and development are described in a separate publication [1].

The method of obtaining velocity measurements in small filaments of the two-phase flows of air and water has been described in a previous publication [2]. Being a basic length/time

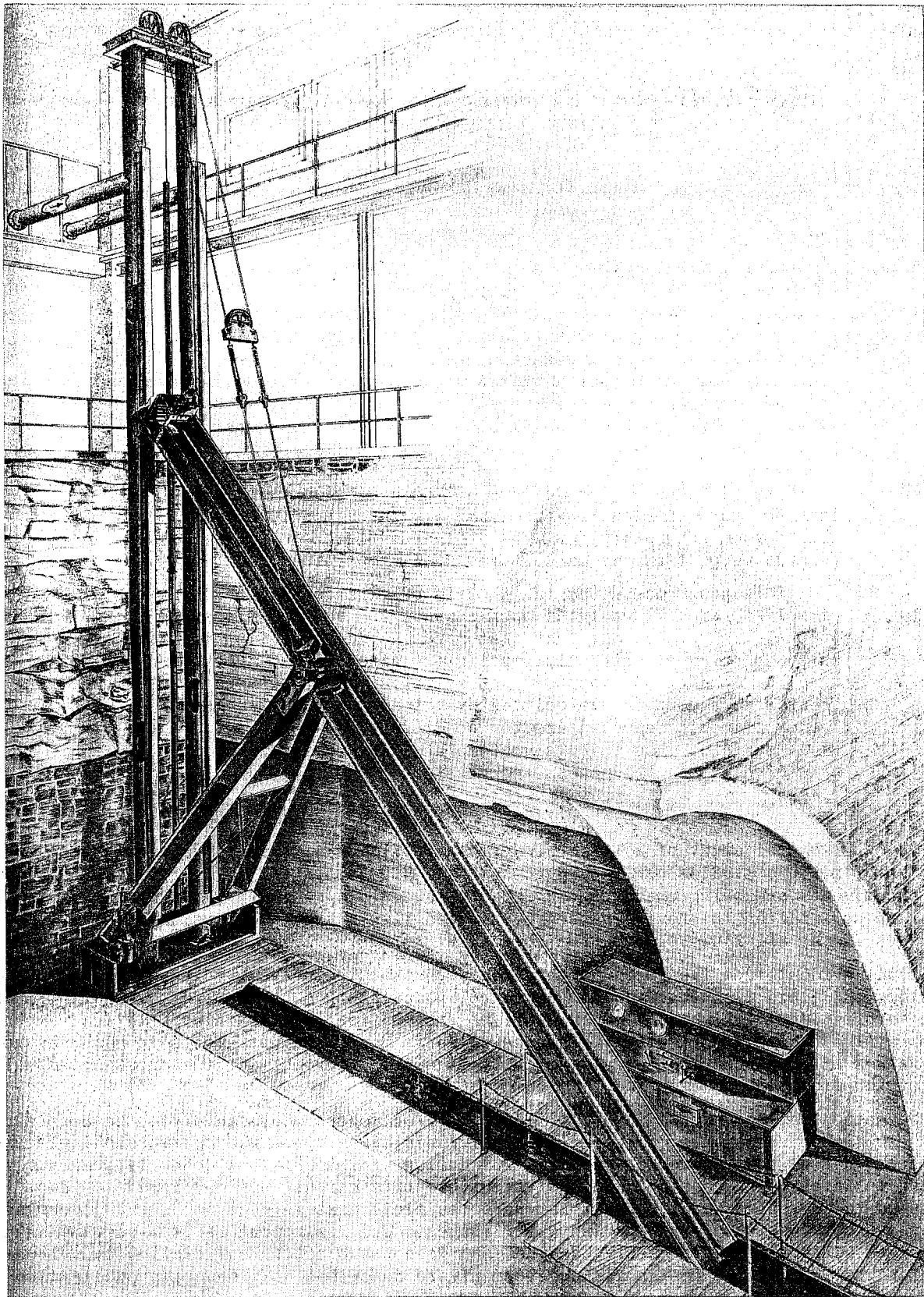


Fig. 1 - Channel for air entrainment study at the St. Anthony Falls Hydraulic Laboratory.

determination over a very short distance (3 to 4 in.), the method required precise injection and timing components. Timing accuracy in the neighborhood of 1/100,000 sec as well as appropriate pickup and amplifying stages were incorporated into a unit electronic circuit, while diesel injection mechanism was employed to achieve the necessary precise identification of the measured filament. The instrument is uniquely fitted for air entrainment studies as it depends neither upon knowledge of the fluid density nor upon condition of strict fluid or mixture homogeneity.

#### DESCRIPTION OF THE FLOW

Ehrenberger [3] in his pioneering work on open-channel aerated flows loosely divided the flow into several zones: water near the floor of the flume, individual air bubbles in water, mixture of water and air, individual drops of water in air, and an overlying movement of air. Several writers subsequently adopted this arbitrary classification as a real delineation between flow phases and attributed special qualities to the intermediate phases to explain observations they had made. Earlier measurements at this laboratory using flow sampling methods to obtain air concentrations [4] showed conclusively that the mean air concentration increased continuously with elevation above the flume bottom. The mean air concentration at the point of measurement closest to the flume floor varied considerably between flows depending upon roughness of the bottom, flow depth, mean flow velocity, and distance from the initiation of aeration. Whatever the value of this lowest measured point, there was established a definite gradient of air concentration with no indication of layer flows or sharply separated flow phases.

The proposal that aeration becomes incipient in a flume or on a spillway when the turbulent boundary layer from the channel bottom intersects the water surface was forwarded by Lane [5]. Later and more elaborate discussions [6, 7] substantiated the hypothesis showing close agreement between calculations and observations for a number of cases.

A characteristic roughening of the water surface immediately before the appearance of "white water" can readily be seen on most free surface installations where self-aeration occurs. Further observations with high-speed photographs such as Fig. 2 illustrate the persistence and magnification of this roughening of the interface into and during the highly aerated condition so that at any instant the aerated flow gives the appearance of exceedingly rough and irregular topography. The depressions or open pockets of air at the interface above the densely underlying fluid and the spikes or projections of water mixture projecting into the air are of varying dimension and spacing in seemingly random occurrence. This violent agitation of the interface between the mixture and the air serves to entrap quantities of air which are then broken into bubbles of a size that presents a balance between the work of agitation of the liquid and the surface energy of the resultant bubbles. Thus, the air bubbles in water are approximately spherical in shape and close to the same general size for a given set of flow conditions. This is illustrated in the photograph of Fig. 3 which is a view through a transparent side wall into a partially aerated flow.

Transport of bubbles further into the flow by turbulence is counteracted by the buoyant force on each bubble so that a balance is reached wherein as much air is buoyed out of the flow as is being drawn into it. This insufflation balance can be compared to the suspended load balance on a reach of a sediment-carrying stream where neither aggradation nor degradation occurs. When the flow has enveloped as many air bubbles as it can retain and when the rough surface topography becomes statistically similar from section to section, the flow can be defined as reaching a condition of normal aeration for the given section, slope, roughness, and discharge.

A portion of the discharge of water through a section in aerated flows consists of water droplets which have become detached from the stream and move through the air by virtue of their initial momentum. Since there is no available mechanism tending to keep them in the air, as there is in the case of air bubbles in the water, they soon fall back into the flow with the result that, on the average, only a very small percentage of the total water discharge flows in this manner at any slopes appreciably less than 90° with the horizontal. Air bubbles in the water tend to follow the turbulent fluctuations of the flow and to have the same mean velocity as the water enveloping them. There is not likely to be any appreciable "slip" in either the upstream or the downstream direction of these isolated bubbles in open channel

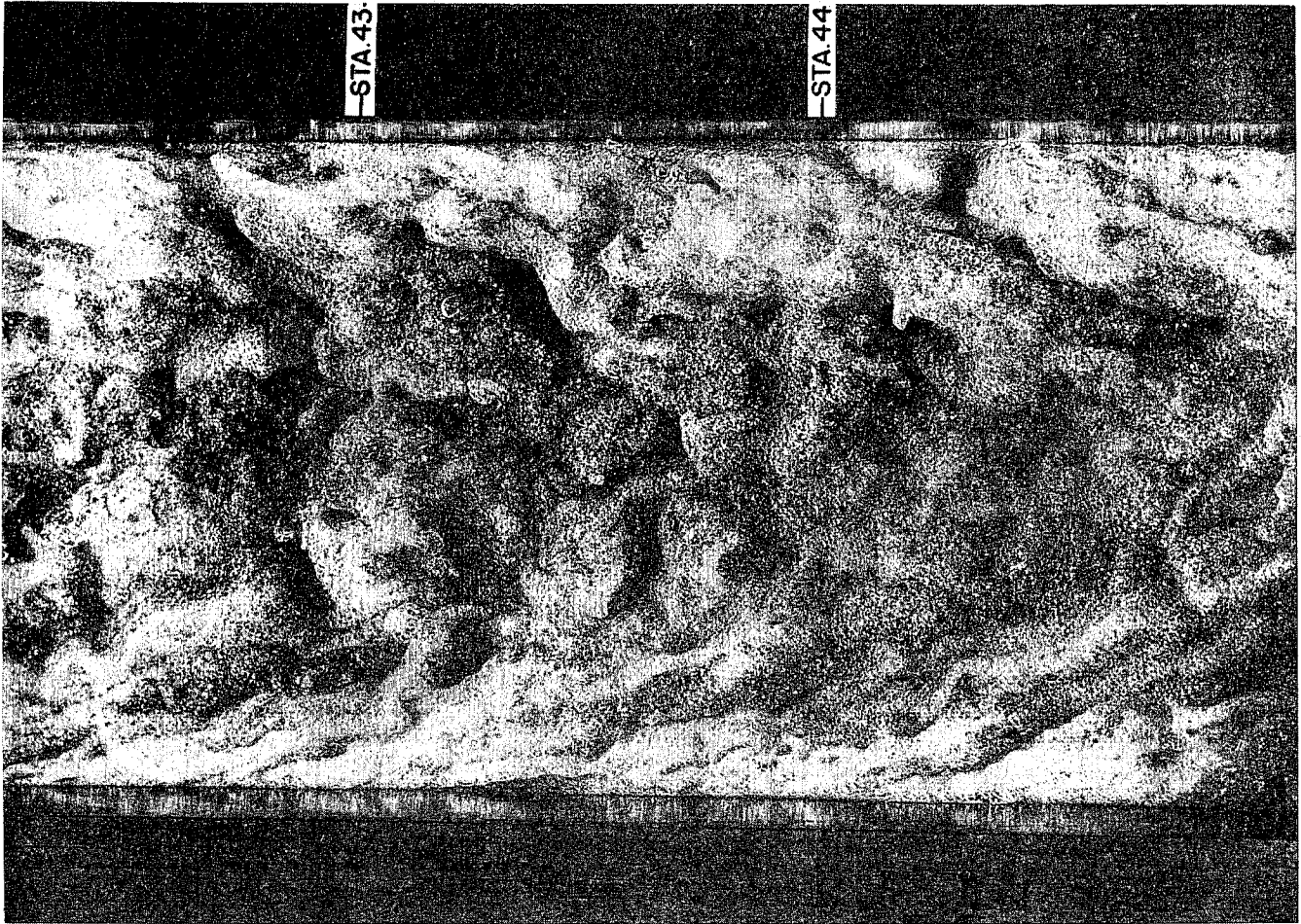


Fig. 2 - Surface of a fully aerated flow at 1/100,000 sec exposure.

flows as compared to the bulk flow velocities. Water droplets moving through the air, however, would have an appreciable velocity relative to the air, which would tend to shorten their trajectories and consequently the time intervals spent as isolated droplets.

The roughened surface and the high flow velocities of the flow mixture tend to drag a considerable volume of air into motion above the flow. A velocity deficiency can be expected in the air-water mixture in the region of the roughened surface because of the drag necessary to sustain this air motion.

At relatively small flow depths, flows on steep gradients break into waves which appear to roll down the channel over a thin underlying stream of water. Bubbles are often entrapped into these wave fronts and the "white water" gives the appearance of regularly spaced bands breasting down the slope. At slightly greater depths where a more general condition of aeration prevails, vestiges of these periodic waves can often be detected by their regularity as slightly higher stages immediately followed by pockets of greater air content. Formation of "roll waves" without consideration of aeration is discussed in a recent summary of the problem [8] which also refers to preceding papers on the same subject.

The variety of differing and, in many cases, opposing opinions expressed by investigators of air entrainment phenomena encountered in a survey of the literature [9] illustrates the

Direction of flow  $\longrightarrow$

many uncertainties that exist in this field. Distributions of density, of velocity, of water discharge, and of momentum flux in a cross section of aerated flows are of interest both for design purposes and ultimately for an understanding of the flows. Bulk increase of section by aeration, comparison of mean velocities with velocities in nonaerated flows, and expectancy of aeration are vital design considerations.

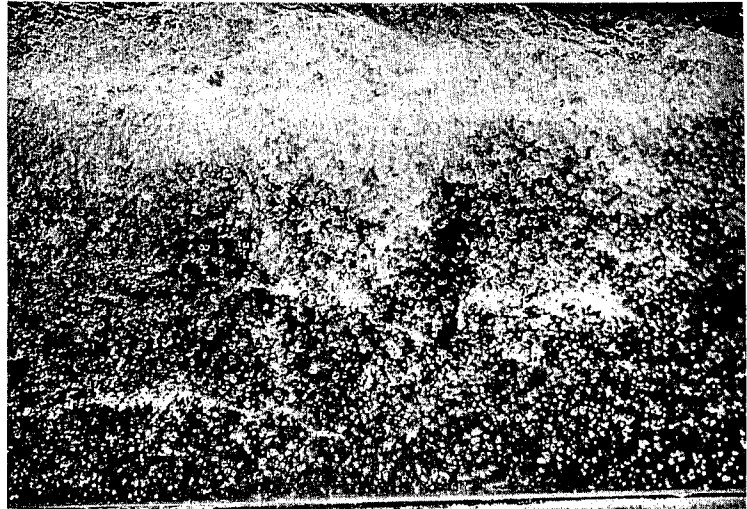


Fig. 3 - Profile of partially aerated flow at 1/50,000 sec exposure.

### EXPERIMENTAL OBSERVATIONS

Experimental measurements described in this paper are largely restricted to velocity and air concentration traverses in flows where normal aeration has been attained. Air concentration  $C$  is defined as the volume of air in a unit volume of air-water mixture. The concept of mixture is intended to embrace both the phase where air bubbles are in the water and the condition where the unevenness of the surface contributes to the variation of density in planes parallel to the flume floor. The component  $x$ ,  $y$ , and  $z$  directions are here taken positive, respectively, in the mean downstream direction of the flow, into the flow from the side wall, and upward normal to the flume floor. The surface of the flow  $\bar{z}$  has been arbitrarily defined as that value of  $z$  where  $C = 0.95$ . This surface envelops between about 98 and 99 per cent of the total water discharge and can still be directly defined by measurement with the concentration instrument. It does not include the bulk of the free air stream, random water droplets flying through the air, and the tips of the highest peaks or protuberances of water extending into the air. The velocity in the positive  $x$  direction is labeled  $u$  and is the only velocity component measured with the SAF velocity meter. Direct velocity measurements can be obtained up to an elevation  $z$  about 1/2 in. smaller than  $\bar{z}$ .

The summation  $\sum_{z=0}^{\bar{z}} (1 - C) \Delta z$  gives the total area occupied by water in a unit breadth of the flow cross section.

The summation  $\sum_{z=0}^{\bar{z}} u (1 - C) \Delta z$  gives the total water discharge  $q_w$  in a unit breadth of the flow cross section.

Mean velocity  $\bar{U}$  of the water through the section is obtained by dividing the second summation by the first, i.e., by directly applying the equation of continuity to the measurements. Mean air concentration  $\bar{C}$  is equal to the summation  $\sum_{z=0}^{\bar{z}} C \Delta z$  divided by  $\bar{z}$  and is seen to be dependent in absolute value upon the choice or definition of  $\bar{z}$ .

Total water discharge  $Q'_w$  in the 1.5-ft wide flume is measured at metering stations in the supply pipes to the equipment. Discharge per unit width  $q'_w = \frac{2}{3} Q'_w$  is used to compare with the integrated water discharge obtained with the instruments in the aerated flows and to act as a control on those measurements.

The inlet of the test flume is designed to present a uniform velocity profile to the jetted inflow at a controlled depth. Inlet velocity is adjusted until it is approximately the terminal velocity for the set slope and discharge. Thus, the flow aeration can be studied independently of large accelerative or decelerative effects and a shorter flume length with a greater degree of flexibility and control can be utilized. Orientation of flume components was selected to give identical inlet geometry throughout the 90° range of flume slopes. Up to

the present time, the equipment has been operated at flume slopes up to  $45^\circ$  with the natural gravity head of 50 ft available at the site; but with completion of a pumped supply system currently being installed, flows with jetted terminal velocities at flume angles approaching  $90^\circ$  will also be tested.

The 1.5-ft breadth of flume is sufficiently large compared to the flow depths used to preserve a two-dimensional region of the flow for several inches to each side of the flume centerline. Complete traverses across the entire cross section of the flume reveal no asymmetry about the centerline for either nonaerated flow, partial aeration, or complete normal aeration. Interpolated lines of equal air concentration drawn between measured values in normal aerated flow at a station 45 ft from the flume inlet are exhibited in Fig. 4. The 0.95

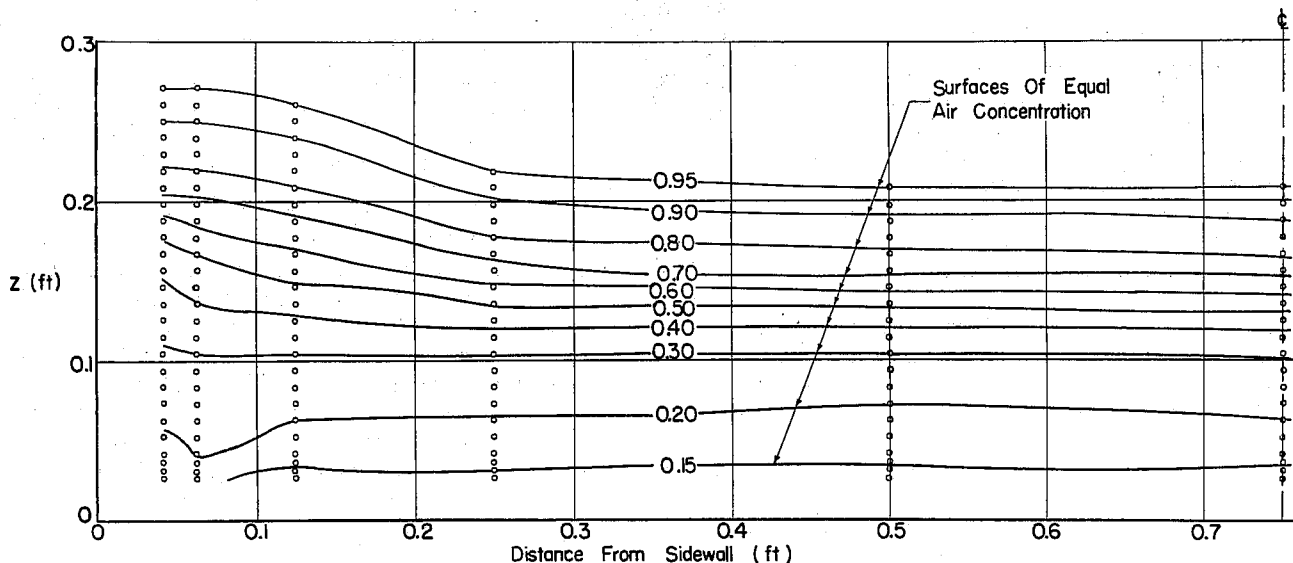


Fig. 4 - Air concentration distribution across section of fully aerated flow.

level of air concentration is considerably higher near the side wall than in the central region of the stream. This side wall effect is especially noticeable before aeration begins in the central region; aeration at the sides is initiated by the turbulence from the side wall boundary at the water surface and thus appears as "white water" sooner. The flat contours in the larger central region of the stream (Fig. 4) illustrate the two-dimensional character of that flow which remains uninfluenced by side wall disturbances and is typical of the centerline region for all flows where complete cross-sectional traverses were obtained. The remaining data presented in this paper were taken in centerline verticals of the stream.

Air concentration verticals at various distances from the inlet gate were used to determine the uniformity of the flow for each set of flow conditions. Figure 5 is a graph of four such verticals taken at 5-ft intervals near the discharge end of the flume for one flow. Although the profiles for stations located 35, 40, and 45 ft from the inlet gate exhibit small differences in the figure, they are definitely closely grouped and, considering measuring variation in positioning and reading the instrument, they cannot be considered as significantly different. On the other hand, the profile for Station 30 is definitely different from the other three to a degree which is typical of changes occurring in the same increment of flume length either in flows with larger discharges or at stations still closer to the inlet where full normal aeration has not been established. For the test data here reported, a condition of normal aeration was substantially reached at Station 45 but not at Station 30. The remaining data presented in this paper were taken in the centerline vertical of the flows 45 ft from the inlet gate and are classified as fully aerated or partially aerated depending upon their similarity to upstream profiles.



Water discharges in the 1.5-ft wide flume of 6.4, 9.6, and 12.8 cfs can be compared for a 15° flume angle by means of the air concentration profiles in Fig. 6 and the velocity profiles in Fig. 7. From Fig. 5 it was determined that the 6.4 cfs flow had reached normal aeration, while respective data for the other flows indicated that the 9.6 cfs flow was approaching normal aeration and that the 12.8 cfs flow was definitely not normal but only partially aerated at Station 45 where it had not yet reached a uniform or normal state.

Observations of these and many other flows with stroboscopic light and in some instances with high-speed photographs such as Fig. 8 indicate that the rapid increase in air concentration gradient begins at an elevation above the flume floor corresponding to the lowest depressions or pockets of air. The air concentration up to this elevation is due to air bubbles in the water, some of which migrate far down into the stream even at low aerations.

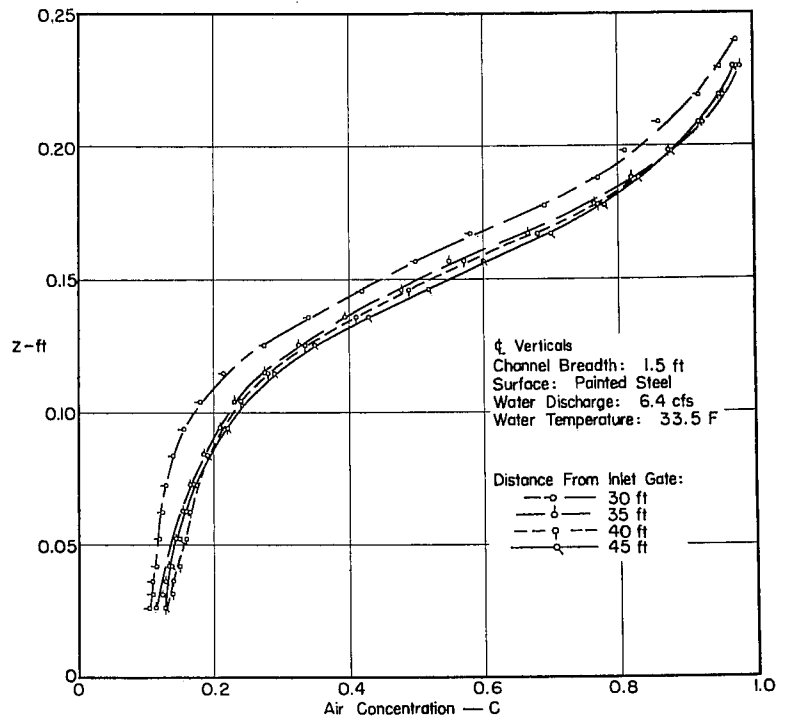


Fig. 5 - Measured air concentration values at various distances from the flume inlet gate.

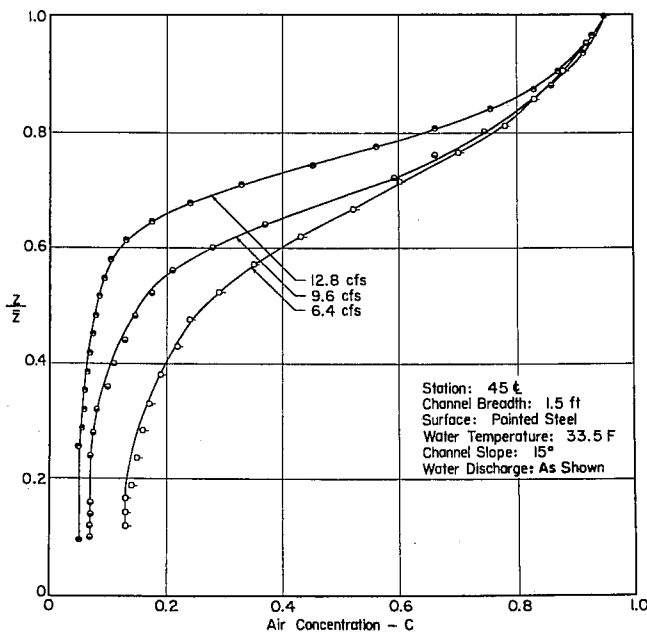


Fig. 6 - Measured air concentration values in aerated flows at various discharges with 15 flume angle.

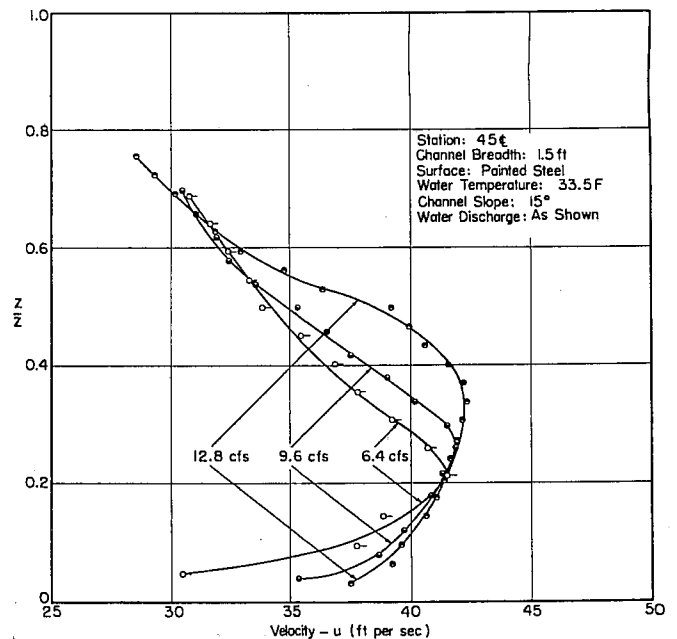


Fig. 7 - Measured velocities in aerated flows at various discharges with 15 flume angle.

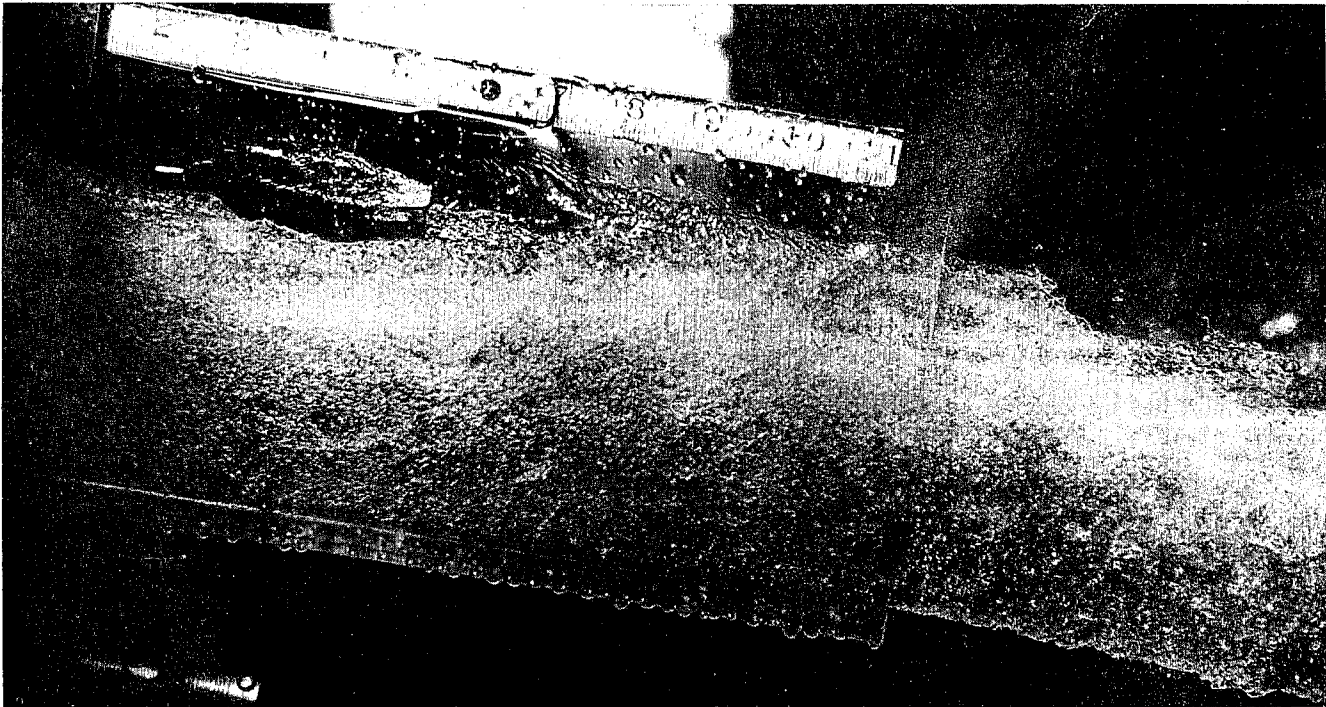


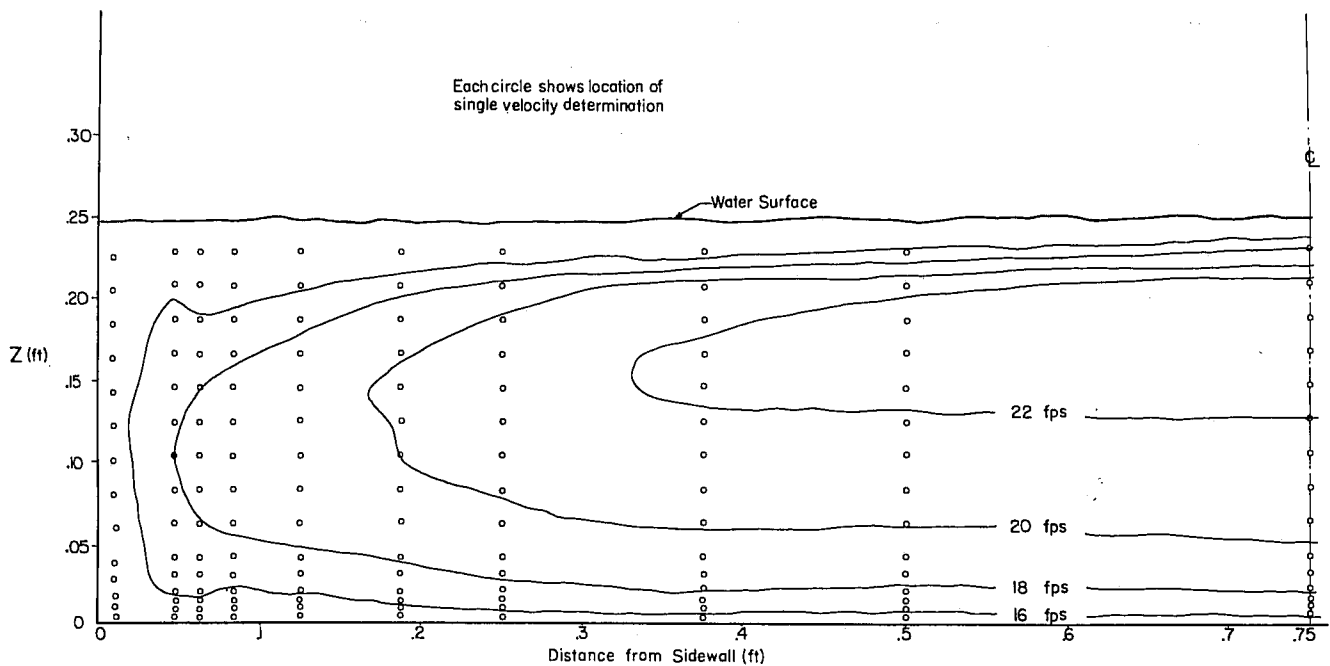
Fig. 8 - Profile of aerated flow through transparent sidewall.

A number of uniform nonaerated flows were set at lower flume angles to form a basis of comparison for the values of mean velocity and for the distributions of discharge and momentum flux obtained with the aerated flows. Typical of these is the condition described by Fig. 9, a nonaerated flow which was set on the same painted steel surface for the 1.5-ft wide test flume as the aerated flows described herein. Manning's  $n$  values for the flume surface were calculated from the basic Pitot velocity measurements and are listed on the figure.

Air concentration and velocity traverses of a series of runs taken at constant discharge on several flume angles are shown in Figs. 10 and 11. The reversal of curvature of the air concentration gradient characteristic of all the data is evident in Fig. 10. The velocity profiles in Fig. 11 show the sharp decline in velocity in the region of rough topography in the upper portions of the streams also characteristic of the various data obtained by direct measurement with the instruments. A tabulation of bulk and mean values of discharge, velocity, and air concentration for the flows of Figs. 6, 7, 10, and 11 is given in Table I. An indication of the control imposed upon the flows and the measurements with the instruments can be had by comparing the independently measured values of incoming water discharges in the table with the water discharges integrated from the air concentration and velocity measurements taken at Station 45 where the flows have become highly aerated. The defined surface elevations listed are directly obtained from the air concentration verticals, and the mean values of air concentration and velocity are obtained with the summations defining these values. The factor  $\alpha$  listed in the table is the coefficient of the mean velocity head which gives the true measure of the kinetic energy of the flow. It is obtained from the following definition equation:

$$\alpha = \frac{\int_{z=0}^{z=\bar{z}} (1 - C) u^3 dz}{(1 - \bar{C}) \bar{U}^3 \bar{z}}$$

where  $\bar{U}$  is the mean velocity obtained in the manner proposed in this paper. It might be pointed out that a mean velocity obtained as a straight spatial average of the measured velocities could, when used in the above expression, give values of  $\alpha$  less than unity which have no significance.



Cross section - Station 45: 45 ft from inlet gate  
 Painted steel flume surface  
 Inlet gate opening: 0.25 ft  
 Water temperature: 34.5 F.  
 Channel slope: 7.75  
 Measured water discharge: 7.1 cfs

Integrated water discharge: 7.044 cfs.  
 Manning's n based on hydraulic radius and entire discharge: 0.0097  
 Manning's n based on two-dimensional discharge at centerline and flow depth: 0.0106  
 Velocity distribution factor for centerline vertical: 1.054

Fig. 9 - Velocity distribution from Pitot traverse in high-velocity channel for non-aerated condition.

#### COMMENTS ON EXPERIMENTAL RESULTS

The measured distribution of air concentration, with the high concentration gradient near the elevation of the original water surface and the lessening of the gradient to either side, are remindful of a number of diffusion problems where at zero time a definite boundary between the two phases exists. The diffusion of water upward into the air by the erupting turbulence and the migration of air bubbles downward into the water, also by a turbulent transport process, can also be regarded as beginning at a zero time with a sharply defined water surface. The skew symmetry of the curves about the elevation of the original surface can be attributed partly to the differences in densities of the materials (water and air) being transported and partly to the presence of the boundary at the flume floor. A mathematical model for similar diffusion problems with heat, momentum, etc., is the Gaussian error function. With the realization, however, that the error function was developed on assumptions applicable mainly to the kinetic theory of the ideal gases, the analogy to highly complicated phenomena of air entrainment wears rather thin. There still remains the problem of defining the distributions analytically with suitable base values for the mathematical expression so that the air concentration can be predicted for any set of flow conditions.

The values of mean air concentration listed in Table I are dependent upon the choice or definition of the flow surface. However, by fixing the flow surface at a definite concentration level, the mean air concentration values assume a relative importance between various

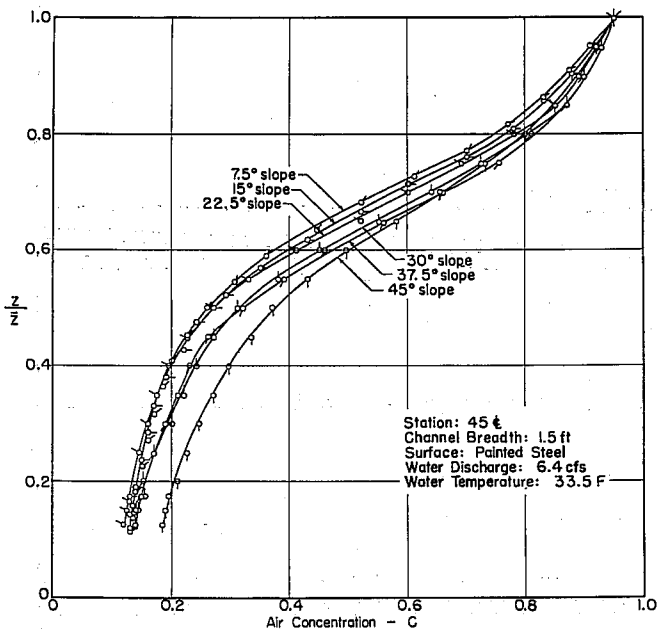


Fig. 10 - Measured air concentration values in full aerated flows at various flume angles with constant discharge.

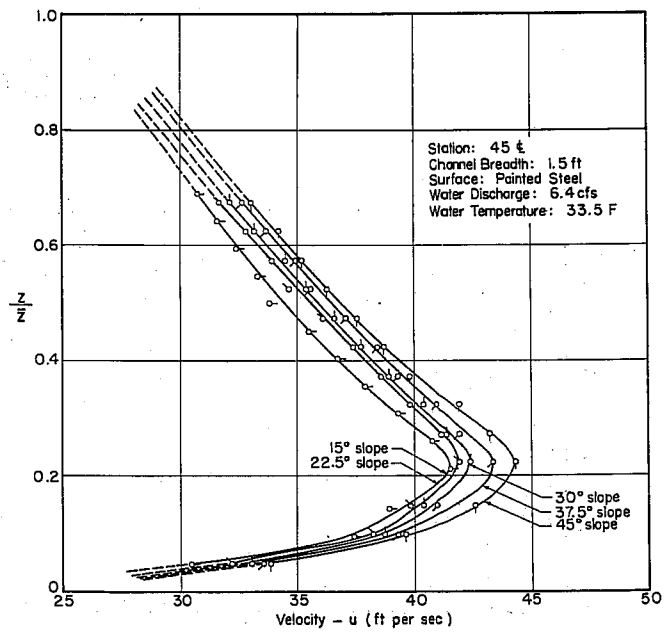


Fig. 11 - Measured velocities in fully aerated flows at various flume angles with constant discharge.

Table I  
Flows with Normal and Partial Aeration on Painted Steel Surface

Flume Angle	Defined Surface Elevation	Measured Water Discharge	Unit Discharge	Integrated Water Discharge	Mean Velocity	Mean Air Concentration	Kinetic Energy Distribution Factor	Remarks
$\theta$	$z$	$Q_w$	$q_w = \frac{2}{3} Q_w$	$q_w$	$\bar{u}$	$\bar{c}$	$\alpha$	
(degrees)	(feet)	(cfs)	(cfs)	(cfs)	(fps)			
7 1/2	0.2292	6.4	4.267	no velocity traverse	-	0.390	-	Normal aeration
15	0.2188	6.4	4.267	4.60	35.38	0.406	1.045	Normal aeration
22 1/2	0.2188	6.4	4.267	4.48	35.93	0.430	1.046	Normal aeration
30	0.2135	6.4	4.267	4.31	36.25	0.443	1.042	Normal aeration
37 1/2	0.2083	6.4	4.267	4.34	37.54	0.445	1.038	Normal aeration
45	0.2032	6.4	4.267	4.27	39.79	0.472	1.035	Normal aeration
15	0.2656	9.6	6.40	6.35	36.22	0.340	1.041	Partial aeration
15	0.3229	12.8	8.53	8.59	36.34	0.268	1.038	Partial aeration

runs for classifying bulking or aeration with such significant variables of slope, discharge, and roughness. An increase in mean air concentration with increasing flume angle is shown in the table for a constant water discharge where the flows were established as having a complete normal aeration. However, the spread of mean air concentration values from 0.39 at 7 1/2° flume angle to 0.472 at 45° flume angle is quite small and suggests that the bulking of the flows is not particularly sensitive to slope changes alone. Since the runs at varying discharges were not all established as reaching a condition of normal aeration, the respective mean air concentration values are not comparable on the basis of different discharges. A comparison of the slope and discharge for the nonaerated run of Fig. 9 with the slope and discharge of the normal aerated run on the 7 1/2° flume angle shows very little difference between the flow variables; yet a bulking of the flow equivalent to a mean air concentration of 0.39 and considerably higher velocities were measured in the aerated flow. It is possible that a slight retardation of the flow at the inlet might have been sufficient to carry the

one flow past a critical where it would then tend to become aerated, but the similarity of traverses at the several downstream stations (similar to Fig. 5) showed no tendency of the flow to revert to a nonaerated condition similar to Fig. 9. It is intended to examine closely this range where aeration may or may not occur for the existence of aeration criticals and also for flows that might have a very low mean air concentration although reaching the condition of normal aeration.

Defined flow depth  $\bar{z}$  experienced very little change with slope for the various runs at constant discharge. Within the narrow range of the reported experiments, any tendency to increase the section by greater bulking appeared to be compensated by a further increase in velocity so that flow depth remained nearly constant.

The relations between slope and velocity in uniform open channel flow in most common usage are empirical modifications of the semi-rational Chezy equation. The Manning form of the equation is familiar to the engineering profession and has been widely accepted for design purposes in conventional mild slope channels. However, doubts have persisted as to the validity of any of the conventional expressions in channels of steep gradients or in cases where high velocities can be expected. With aeration also a consideration, the problem of predicting mean velocity at steep gradients becomes much more complicated.

Earlier authors on the subject of air entrainment [10, 11] have proposed computed values of the hydraulic radius to be used with conventional  $n$  values. In one case, the computed value of hydraulic radius is based on an area and the wetted perimeter of a computed shallower section occupied by water alone, and in the other case the computed value is based on the area of the shallower section of water alone and a wetted perimeter of the full flow section. Checking these expressions with the data presented in Table I,  $n$  values are obtained which show no agreement with the  $n$  value given in Fig. 9 for uniform flow in the flume with no aeration.

Combining the Manning equation for a unit breadth of channel with the equation of continuity, an expression for velocity in terms of discharge, slope, and roughness value is obtained:

$$V = \left( \frac{1.486}{n} \right)^{0.6} q_w^{0.4} S^{0.3}$$

Using the measured value of  $q_w = 4.267$  from Table I and  $n = 0.0106$  from Fig. 9, the following values of mean velocity are obtained for flume angles from 15° to 45°:

Flume Slope	15°	22-1/2°	30°	37-1/2°	45°
Mean Velocity by Manning Formula, V fps	23.1	26	28.1	29.8	31.2
Mean Velocity measured (from Table I)	35.38	35.93	36.25	37.54	39.79

By comparing these predicted mean velocities from the Manning equation with the measured  $\bar{U}$  from Table I, it can be seen that in all cases the predicted velocity is too low.

The importance of increase of section or hydraulic radius because of aeration in explaining the high values of mean velocity must be minimized because it can be seen from the velocity profiles in Figs. 7 and 11 that any increase in section is more than offset by the upper surface drag on the overlying air, which causes a deficiency in velocity below the mean velocity throughout the added area.

Figure 12 illustrates the difference between cumulative water discharges in the flow section for fully aerated and nonaerated flows. It will be observed that for the test conditions recorded, nearly 90 per cent of the water discharge is in the lower 0.6 depth in the case of the aerated flow.

Figure 13 is a generalized diagram of the velocity and air concentration curves in aerated flow modeled from the curves which were obtained by direct measurement. The position of the arbitrarily defined flow surface with respect to the whole flow field is indicated by the dotted line. It should be pointed out that the portion of the velocity curve above the defined surface refers to the moving air mass and not to the scattered water droplets which would be expected to have velocities closer to the velocities of the lower flow regions from which they were ejected. The general shape of these curves is believed to be characteristic

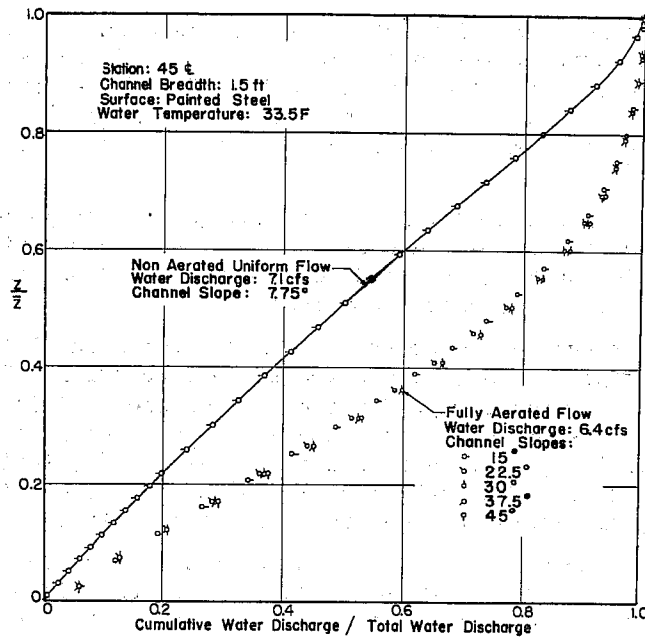


Fig. 12 - Dimensionless representation of the cumulation of water discharge with elevation above flume bottom.

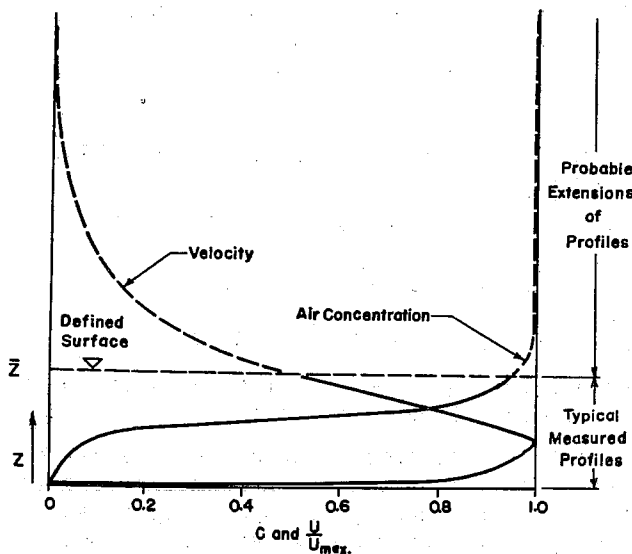


Fig. 13 - Profile of velocity and air concentration in the entire flow field of air and water.

of fully aerated flows although details of the profiles may vary with depth, roughness, and slope. Scale effect in extremely large flows could possibly rule out the support of as large a rough topography region relative to the total depth as was found in the flows measured.

#### ACKNOWLEDGMENT

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1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities.

2. It then outlines the various methods and techniques used to collect and analyze data, including interviews, surveys, and focus groups.

3. The next section describes the process of identifying and defining the research objectives and questions, as well as the selection of appropriate samples.

4. This is followed by a detailed discussion of the data collection procedures, including the design of questionnaires and the implementation of field studies.

5. The final part of the document covers the analysis and interpretation of the collected data, highlighting the challenges and limitations of this process.

6. In conclusion, the document emphasizes the need for a systematic and rigorous approach to research, ensuring the reliability and validity of the findings.

7. The document also provides a list of references and resources for further study, as well as a glossary of key terms and concepts.