Supplementary material for New constraints on equatorial temperatures during a Late Neoproterozoic snowball Earth glaciation

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Supplementary Material: Model of ground temperature

Here we derive a model that describes the diurnal and annual cycle in temperature at the Cattle Grid Mine site as a function of its paleolatitude during a snowball glaciation. Both geological observations of the sand wedges (Fig. 2) and sedimentary structures indicate the presence of an active aeolian sand sheet and dunes during the development of the wedges, implying that the surface was bare throughout the year. The model describes the temperature at the surface and in the regolith below assuming snow-free surface conditions.

The zonal-mean climate during a snowball glaciation differed from modern conditions due to factors including a smaller thermal inertia associated with ice covering the oceans and a higher surface albedo (Pierrehumbert et al., 2011). Simulations of a snowball glaciation using comprehensive climate models vary substantially depending on how quantities such as surface albedo are parameterized (e.g., Fig. 5 in Pierrehumbert et al., 2011). Here we adopt a simpler approach and assume that the amplitudes of the diurnal and annual cycles in temperature at a location with bare land during a snowball glaciation would be similar to a location with bare land at the same latitude in the modern climate, which is consistent with the range of results for the annual cycle from comprehensive climate models (Pierrehumbert et al., 2011). In order to describe the characteristic meridional structure of modern observed surface temperature variability over land, we use a combination of observed temperature variability and the most salient physics involved with periodic variability that arises from a linear response to solar insolation, drawing on previous energy balance models of Earth's climate (North et al., 1981). This approach focuses only on periodic temperature variability, rather than including a representation of the annual-mean temperature, and it treats the vertical structure in the atmosphere, horizontal energy transport, the planetary albedo, the dependence of outgoing infrared radiation on surface temperature, and the effective heat capacity associated with surface temperature changes as constant. Scalar parameters that describe the heat capacities associated with annual and diurnal variability in the model are derived from observations of modern temperature variability.

Solar insolation

The analytical formula for insolation reaching the Earth as a function of latitude and season is derived in a number of textbooks (e.g., Sec. 2.7 of Hartmann 1994 and Sec. 7.3 of Pierrehumbert 2011), and the relevant points for this discussion are summarized below. We consider a circular orbit because the eccentricity of Earth's orbit is expected to be negligible for the purposes of this model. In this case, the insolation is

$$S(t) = \begin{cases} S_c[\sin\phi \sin\delta + \cos\phi \cos\delta \cos h] & -h_0 < \mathrm{mod}(h+\pi, 2\pi) - \pi < h_0 \text{ [day]} \\ 0 & \text{otherwise [night]} \end{cases} . \tag{1}$$

Here $S_c = 1285\ \mathrm{Wm}^{-2}$ is the solar constant (94% of its present value), ϕ is latitude, δ is the declination angle (latitude of the point on Earth's surface directly under the sun at noon), h is the hour angle (longitude of the subsolar point relative to its position at noon), and h_0 is the hour angle at sunset (with $-h_0$ being sunrise). The declination angle is computed from

$$\sin(\delta) = \sin(\varepsilon)\sin(\lambda) \tag{2}$$

with being obliquity and λ being the solar longitude of Earth's orbit (angle between vernal equinox and the position in Earth's orbit). We use the modern value of obliquity by default, $\varepsilon = 23.4^\circ$, and we also consider a high-obliquity world with $\varepsilon = 54°$. For a circular orbit, λ varies at a uniform rate during the course of the year. The hour angle at sunset is computed from

$$\cos(h_0) = -\tan(\phi)\tan(\delta) \tag{3}$$

except when $|\phi| \geq \frac{\pi}{2} - |\delta|$, which occurs during polar day and polar night. In this case, when ϕ and δ have the same sign (polar day) we use $h_0 = \pi$ and otherwise (polar night) we use $h_0 = 0$. We use a measure of time (t) that is normalized by the diurnal period, i.e., we measure time in days. In this case, the hour angle is $h = 2\pi t$ and the solar longitude is $\lambda = 2\pi t/365$. This fully specifies the insolation $S(t)$ at each latitude ϕ.

Next, we separate the insolation into discrete frequency components. For each latitude, we numerically generate a one-year time series of $S(t)$ at N evenly-spaced times. The mean value is

$$S_0 = \frac{1}{N}\Sigma S(t) \tag{4}$$

and we calculate each sinusoidal component n with period P_n using the Fourier transform,

$$Y_n = \frac{2}{N}\Sigma e^{2\pi i t/P_n} S(t), \tag{5}$$

where the factor of 2 accounts for both positive and negative frequencies. The amplitude (S_n) and phase (θ_n) of each Fourier component is given by

$$S_n = |Y|,\ \theta_n = \tan^{-1}\left[\frac{\mathrm{Im}(Y)}{\mathrm{Re}(Y)}\right]. \tag{6}$$

We isolate the annual ($P_1 = 365$), semiannual ($P_2 = 365/2$), and diurnal ($P_3 = 1$) components as the dominant frequencies of variability. Neglecting all other frequencies, the insolation is hence approximated as

$$S(t) \approx S_0 + S_1\cos\left(\frac{2\pi}{P_1}t - \theta_1\right) + S_2\cos\left(\frac{2\pi}{P_2}t - \theta_2\right) + S_3\cos\left(\frac{2\pi}{P_3}t - \theta_3\right). \tag{7}$$

In Fig. 8a,c, we plot the range of each frequency of net solar forcing, $2(1-\alpha_p)S_n$, as a function of latitude, where α_p is the planetary albedo. Diurnal variability dominates in low to mid latitudes and annual variability dominates in high latitudes. The semiannual component is important near the equator, where the sun passes overhead twice per year, as well as in high latitudes where polar night causes the seasonal variability to depart substantially from an annual sinusoid. The annual cycle dominates over the semiannual cycle in the latitudes of interest for this study (7°–14°), and this dominance is felt even more strongly at depth in the sandy regolith due to longer period variability penetrating more deeply (as described below). Hence for simplicity we focus here only on the annual and diurnal components of variability in solar radiation, S_1 and S_3.

Surface temperature

We let the annual and diurnal frequency components of the surface temperature evolution, T_n, evolve as a linear function of the net solar forcing, $(1-\alpha_p)S_n$. The proportionality constant is chosen based on modern observations of surface temperature variability over land. The physical basis for this relationship is discussed below.

We consider the evolution of the surface temperature (T_s) subject to solar insolation of a single frequency superimposed on the annual mean. Assuming a single column with no horizontal energy transport in the atmosphere, the surface temperature evolution can be approximated as

$$c_n \frac{dT_s}{dt} = (1-\alpha_p)\left[S_0 + S_n \cos\left(\frac{2\pi}{P_n}t - \theta_n\right)\right] - \left[L_m + L_T\left(T_s - T_m\right)\right]. \tag{8}$$

Here c_n is the heat capacity associated with changes in T_s on the timescale indicated by n. Since we are interested in the temperature variability at an ice-free location, we use an approximation of the ice-free modern planetary albedo based on satellite observations (Graves et al., 1993), $\alpha_p = 0.2 + 0.36\sin^2(\phi)$. The parameters L_m and L_T represent the infrared radiation to space as a linear function of the surface temperature, with T_m being the melting point; this approximation is commonly made in simple climate models (e.g., North et al., 1981). The linear system (8) can be solved exactly as

$$T_s = T_m + \frac{(1-\alpha_p)\,S_0 - L_m}{L_T} + T_n \cos\left(\frac{2\pi}{P_n}t - \tilde{\theta}_n\right) \tag{9}$$

with phase

$$\tilde{\theta}_n = \theta_n + \tan^{-1}\frac{2\pi c_n}{P_n L_T} \tag{10}$$

and amplitude

$$T_n \equiv \frac{(1-\alpha_p)S_n}{\sqrt{L_T^2 + (2\pi c_n/P_n)^2}}. \tag{11}$$

Here we have neglected the transient term in the solution to eq. (8) associated with the decay of the initial condition. Annual and diurnal amplitudes (T_n) can be considered together in this framework by linear superposition of the solutions for the two frequencies.

In principle, the entire denominator in eq. (11) is adjusted to match temperature variability over land in modern observations. This is equivalent to the more physically intuitive approach of letting $L_T = 2\ \mathrm{Wm^{-2}K^{-1}}$, which is a typical value used in simple climate models (e.g., North et al., 1981), and adjusting the parameter c_n which includes the heat capacity of the sandy regolith and the atmospheric column above. Due to the dependence of the diffusive penetration depth on the frequency of the forcing (as described in the following section), the diurnal cycle is not expected to have the same effective heat capacity as the annual cycle. Using the latter approach, we choose values of c_n for the diurnal and annual cycles to match the amplitudes of variability over land in modern observations (New et al., 1999) when the solar constant in the model is set to the modern value ($S_0 = 1370\mathrm{Wm^{-2}}$), which leads to $c_1 = 3\times 10^7\mathrm{Jm^{-2}K^{-1}}$ for the annual cycle and $c_3 = 0.1\times 10^7\mathrm{Jm^{-2}K^{-1}}$ for the diurnal cycle. This set of parameters fully specifies the amplitudes of diurnal and annual cycles in surface temperature, T_n, which will be used in the following section to force variability within the regolith below. In Fig. 8b,d we plot the surface temperature annual range ($2T_1$) and diurnal range ($2T_3$) as a function of latitude.

We note that the planetary albedo above ice-free columns may likely have been different during a snowball glaciation compared with today due to differences in clouds and atmospheric composition. Similarly, the heat capacity of the atmospheric column may have been reduced due to the lower humidity. However, the dramatic differences in penetration depth between annual and diurnal forcing (Fig. 8) is expected to dominate over these factors.

Thermal diffusion in sandy regolith

We consider the ground as a one-dimensional thermally diffusive medium of infinite depth with sinusoidally-varying surface temperature specified from eq. (9)-(11). An alternative approach would have been to use a single thermodynamic model of the column of ground below an atmospheric column, but we choose to model the atmosphere–surface system separately (discussed above) as a forcing on the subsurface ground in order to more transparently match the surface temperature variability with modern observations. In the interior ($0 \le z < \infty$), we model the temperature field $T(z,t)$ to evolve according to

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}, \tag{12}$$

with the surface boundary condition

$$T(0,t) = T_n \cos\left(\frac{2\pi}{P_n} t - \tilde{\theta}_n\right) \tag{13}$$

using T_n from eq. (11). A solution to this system is

$$T(z,t) = T_n e^{-z/\lambda} \cos\left(\frac{2\pi}{P_n} t - \tilde{\theta}_n - \frac{z}{\lambda}\right) \tag{14}$$

with

$$\lambda \equiv \sqrt{\frac{2\kappa P_n}{2\pi}}. \tag{15}$$

Note that here, as above, the response to the total forcing can be calculated using linear superposition of the separate frequencies. Using a sandy regolith thermal diffusivity of $\kappa = 1.1 \times 10^{-6}$ m^2/s as in previous work (Maloof et al., 2002), this leads to $\lambda = 0.17$m for diurnal variability and $\lambda = 3.3$m for annual variability. These values are shown with one significant figure in the main text. Note that thermal diffusivity values for sandstone are approximately the same as those used here, being $\kappa \approx 1.12 \times 10^{-6}$ m^2/s.

Heat transport into permafrost is typically modeled as a diffusive process (Riseborough et al., 2008). However, once melting occurs at the surface, advection of heat by liquid water (Rowland et al., 2011) could make our estimates of seasonal ground temperature fluctuations at depth conservative. Heat is also lost by the ground due to latent heating during the melt of the pore ice and gained during the freeze of the pore water, which would reduce our diffusive estimates of seasonal ground temperature fluctuations at depth by a factor that depends on the concentration of ice in the regolith. Note that these processes may also influence the value of c_n in eq. (11). The presence of sand wedges rather than ice wedges, as well as the aeolian sand sheet, suggests an arid paleoenvironment where ground ice concentrations, guided by modern observations (Berg and Black, 1966; Bockheim et al., 2007; Campbell et al., 1998), may have been only a few percent.

In order to get an approximate scaling for the influence of latent heat of phase changes within the regolith, we consider a phase boundary between liquid and solid that migrates vertically in a thermally diffusive medium in response to a change in the surface temperature, which is a Stefan problem. When the surface temperature above a frozen column of pure material is suddenly warmed to a temperature ΔT above the melting point, the liquid-solid interface will propagate downward according to

$$h = \sqrt{\frac{2\kappa c_p \Delta T t}{L}}, \tag{16}$$

where h is the depth of the interface, c_p is the specific heat capacity above the solid-liquid interface, L is the latent heat of fusion, and t is time (Worster, 2000). The solution (16) requires the approximation that the Stefan number, $S \equiv L/(c_p \Delta T)$, is large. Here we use the Stefan solution (16) to estimate the depth of thaw penetration, which is a standard procedure in permafrost modeling (Riseborough et al., 2008). We use a latent heat of fusion of $L = \gamma L_0$, where $L_0 = 3 \times 10^2$ J/g is the value for pure ice and γ is the fraction of the ground that is ice rather than sand. We use $c_p = 1$ J/g/K for the heat capacity of the sand/water mixture above the phase interface, which is a typical value for sand. We consider several values for the seasonal ground surface positive degree days, $\Delta T t$. For a 14° paleolatitude with annual-mean temperature of 0°C, $\Delta T t = T_1(1\text{yr})/\pi = 2.4$K yr, where $T_1 = 7.5$K is the surface temperature annual cycle amplitude (half the annual range plotted in Fig. 8b) and we have used that $\int_0^{0.5} \sin 2\pi x dx = 1/\pi$. Solving eq. (16) for γ, we find that in order for the thaw to penetrate to a depth of at least $h = 4m$ with these parameter values, the ice content of the regolith would have to be less than $\gamma = 0.03$. For a 14° paleolatitude with annual-mean temperature of 2.5°C, integration of the positive values of the vertically-shifted sinusoidal surface ground temperature yields $\Delta T t = 1.3$K yr, which corresponds to an ice content of $\gamma = 0.02$ for 4m thaw penetration. For a 7° paleolatitude with annual-mean temperature of 0°C, $\Delta T t = T_1(1\text{yr})/\pi = 1.2$K yr using $T_1 = 3.9$K from Fig. 8b, and $\gamma = 0.02$ for 4m thaw penetration. This implies that if only diffusion causes heat transport into the ground, the ice content of the regolith would have to be a few percent or less in order to allow periodic melting from the surface down to 4m depth driven by the seasonal surface temperature variations we calculate for 7° to 14° paleolatitude, with lower required ice contents for annual-mean temperatures farther from 0°C. Advection by liquid water, however, would allow for the possibility of larger ice contents within the ∼4m freeze-thaw zone.

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